Methods for Solving Large Scale Problems of Customer Order Scheduling

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Problem Statement

Input data

m is the number of customers

n is the number of products

$p_{ij} \geq 0$ is the duration of producing product j for customer i

$s_{jj'} \geq 0$ is the setup time from product j to product j'

s'_j is the initial setup to product j

d_i is the due date for customer i

q_i is the weight for customer i

Solution representation

We define operation as a pair of customer and product type (i, j), $i = 1, \dots, m$, $j = 1, \dots, n$. Operations should be scheduled without preemptions. A set of feasible solution Π consists of permutations of operations.

Criteria

Total completion time

$\sum_{i=1}^{m} C_i$

Weighted throughput

$\sum_{i \in O(\pi)} q_i$
$O(\pi)$ is the subset of customers, for which $C_i \leqslant d_i$ in π

Makespan

$\max_{i=1,\ldots,m} C_i$

Maximum lateness

$\max_{i=1,\ldots,m}\{C_i - d_i\}$

Previous Research

Hazir, O., Gunalay, Y., Erel, E. (2008). Customer order scheduling problem: a comparative metaheuristics study. The International Journal of Advanced Manufacturing Technology, 37, 589-598.

Erel, E., Ghosh, J. B. (2007). Customer order scheduling on a single machine with family setup times: Complexity and algorithms. Applied Mathematics and Computation, 185(1), 11-18.

Cetinkaya, F. C., Yeloglu, P., Catmakas, H. A. (2021). Customer order scheduling with job-based processing on a single-machine to minimize the total completion time.

Kovalenko, Y. V., Zakharov, A. O. (2020). The Pareto set reduction in bicriteria customer order scheduling on a single machine with setup times. In Journal of Physics: Conference

Previous Research

B.M.T. Lin, A.V. Kononov (2007) Customer order scheduling to minimize the number of late jobs, Euro J Oper Res

Bruno de Athayde Prata, Carlos Diego Rodrigues, Jose Manuel Framinan (2021) Customer order scheduling problem to minimize makespan with sequence-dependent setup times, Comput & Ind Engin

Z. Shi, L. Wang, P. Liu and L. Shi (2017) Minimizing Completion Time for Order Scheduling: Formulation and Heuristic Algorithm, IEEE Transactions on Automation Science and Engineering

Jose M. Framinan, Paz Perez-Gonzalez (2017) New approximate algorithms for the customer order scheduling problem with total completion time objective, Comp & Oper Res

Mathematical Programming Model 1

Operation o is a pair (i, j).
We denote the set of all operations as O.

$$x_{ok} = \begin{cases} 1, & \text{if operation } o \text{ is placed in position } k, \\ 0 & \text{otherwise,} \end{cases}$$

$t_k^f \geq 0$ is the completion time of an operation in position k,

$T_i^f \geq 0$ is the completion time of the service of customer i,

$$k \in N, \; o \in O, \; i \in M.$$

Mathematical Programming Model 1 (constraints)

Criterion. The sum of completion times

$$\sum_{i=1}^{m} T_i^f \to \min \tag{1}$$

Constraints

$$\sum_{k=1}^{nm} x_{ok} = 1, \; o \in O, \tag{2}$$

$$\sum_{o \in O} x_{ok} = 1, \; k \in N, \tag{3}$$

$$t_1^f \geq \sum_{o \in O} x_{o1}(p_o + s'_o), \tag{4}$$

$$t_k^f \geq t_{k-1}^f + p_o + \sum_{o' \in O} x_{o',k-1} s_{o'o} - H(1 - x_{ok}), \tag{5}$$

$$k = 2, \ldots, nm, \; o \in O,$$

$$T_i^f \geq t_k^f - H(1 - x_{ok}), \; k \in N, \; o \in O, \tag{6}$$

$$T_i^f \geq 0, \; t_k^f \geq 0, \; x_{ok} \in \{0, 1\}, \; y_i \in \{0, 1\}, \; i \in M, \; k \in N, o \in O. \tag{7}$$

Mathematical Programming Model 2

$$x_{ijk} = \begin{cases} 1, & \text{if product } j \text{ of customer } i \text{ is placed in position } k, \\ 0 & \text{otherwise;} \end{cases}$$

$$b_{j'jk} = \begin{cases} 1, & \text{if arise a setup between products } j' \\ & \text{ and } j \text{ in position } k, \\ 0 & \text{otherwise,} \end{cases} \qquad k = 2, \dots, nm, \ j', j \in J, j' \neq j;$$

$$z_{ik} = \begin{cases} 1, & \text{if customer } i \text{ is in the system in position } k, \\ 0 & \text{otherwise,} \end{cases} \ k \in N, \ i \in M$$

$$U_{ijki'} = \begin{cases} 1, & \text{if both } x_{ijk} = 1 \text{ and } z_{i'k} = 1, \\ 0 & \text{otherwise,} \end{cases} \ k \in N, \ j \in J, \ i', i \in M;$$

$$V_{j'jki'} = \begin{cases} 1, & \text{if both } b_{j'jk} = 1 \text{ and } z_{i'k} = 1, \\ 0 & \text{otherwise,} \end{cases} \ k \in N, \ j', j \in J, \ i' \in M;$$

Mathematical Programming Model 2 (constraints)

$$\sum_{i'=1}^{m}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{nm} p_{ij}U_{ijki'} + \sum_{i'=1}^{m}\sum_{j'=1}^{n}\sum_{j=1}^{n}\sum_{k=2}^{nm} s_{j'j}V_{j'jki'} + m\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij1}s'_j \to \min, \tag{8}$$

$$\sum_{k=1}^{nm} x_{ijk} = 1,\ i \in M,\ j \in J, \quad \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = 1,\ k \in N, \tag{9}$$

$$kx_{ijk} \le a_i,\ i \in M,\ j \in J,\ k \in N, \tag{10}$$

$$\sum_{k=1}^{nm} z_{ik} = a_i,\ i \in M, \tag{11}$$

$$z_{i,k+1} \le z_{ik},\ i \in M,\ k = 1, \dots, nm-1, \tag{12}$$

$$\sum_{i=1}^{m} x_{ijk} + \sum_{i=1}^{m} x_{i,j',k-1} - 1 \le b_{j'jk},\ j, j' \in J,\ k \in N, \tag{13}$$

$$x_{ijk} + z_{i'k} - 1 \le U_{ijki'},\ i, i' \in M,\ j \in J,\ k \in N, \tag{14}$$

$$b_{j'jk} + z_{i'k} - 1 \le V_{j'jki'},\ i' \in M,\ j, j' \in J,\ k \in N, \tag{15}$$

$$x_{ijk} \in \{0,1\},\ y_i \in \{0,1\},\ a_i \in \{0,1\},\ n \le a_i \le nm, \tag{16}$$

$$b_{j'jk} \in \{0,1\},\ z_{ik} \in \{0,1\},$$

$$U_{ijki'} \in \{0,1\},\ V_{j'jki'} \in \{0,1\},\ i, i' \in M,\ j, j' \in J,\ k \in N.$$

Mathematical Programming Model 3

Operation o is a pair (i, j).
We denote the set of all operations as O, and let O_i corresponds to operations of customer $i \in M$. We also add two auxiliary operations 0 and $nm+1$ of zero duration. $O' := O \cup \{0, nm+1\}$.

$$\bar{x}_{ol} = \begin{cases} 1, & \text{if operation } o \text{ immediately proceeds operation } l, \\ 0 & \text{otherwise,} \end{cases}$$

$t_o^f \geq 0$ is the completion time of operation o,
$T_i^f \geq 0$ is the completion time of producing the last product of customer i,
$o, l \in O',\ i \in M.$

Mathematical Programming Model 3 (constraints)

Criterion. The sum of completion times

$$\sum_{i=1}^{m} T_i^f \to \min \tag{17}$$

Constraints

$$\sum_{l \in O' \setminus \{0\}} \bar{x}_{ol} = 1, \; o \in O' \setminus \{nm+1\}, \tag{18}$$

$$\sum_{o \in O' \setminus \{nm+1\}} \bar{x}_{ol} = 1, \; l \in O' \setminus \{0\}, \tag{19}$$

$$t_o^f \geq t_l^f + s_{lo} + p_o - H(1 - \bar{x}_{lo}), \; o, l \in O', \tag{20}$$

$$T_i^f \geq t_o^f, \; o \in O_i, \; i \in M, \tag{21}$$

$$T_i^f \geq 0, \; t_o^f \geq 0, \; \bar{x}_{ol} \in \{0,1\}, \; y_i \in \{0,1\}, \; i \in M, \; o, l \in O'. \tag{22}$$

$$\bar{x}_{ol} = \begin{cases} 1, & \text{if operation } o \text{ immediately proceeds operation } l, \\ 0 & \text{otherwise,} \end{cases}$$

$$z_{ol} = \begin{cases} 1, & \text{if operation } o \text{ proceeds operation } l, \\ 0 & \text{otherwise,} \end{cases}$$

$$w_{olo'} = \begin{cases} 1, & \text{if } z_{lo'} = 1 \text{ and } x_{ol} = 1, \\ 0 & \text{otherwise,} \end{cases}$$

$o, o', l \in O'$.

Mathematical Programming Model 4 (constraints)

Criterion. The sum of completion times

$$\sum_{i=1}^{m} T_i^f \to \min \tag{23}$$

Constraints

$$\sum_{l \in O' \setminus \{0\}} \bar{x}_{ol} = 1, \; o \in O' \setminus \{nm+1\}, \tag{24}$$

$$\sum_{o \in O' \setminus \{nm+1\}} \bar{x}_{ol} = 1, \; l \in O' \setminus \{0\}, \tag{25}$$

$$t_{o'}^f = \sum_{o,l \in O'} w_{olo'} s_{ol} + \sum_{l \in O'} z_{lo'} p_l + p_{o'}, \; o' \in O', \tag{26}$$

$$w_{olo'} \geq z_{lo'} + \bar{x}_{ol} - 1, \; o, l, o' \in O', \tag{27}$$

$$z_{o,o'} + z_{o',o} = 1, \; o', o \in O', \; o' \neq o, \tag{28}$$

$$x_{o,o'} + \bar{x}_{o',o} \leq 1, \; o', o \in O', \; o' \neq o, \tag{29}$$

$$\sum_{o \in O} z_{oo'} - \sum_{o \in O} z_{ol} + (nm-1)\bar{x}_{o'l} \leq nm, \; o', l \in O. \tag{30}$$

$$T_i^f \geq t_o^f, \; o \in O_i, \; i \in M, \tag{31}$$

$$T_i^f \geq 0, \; t_o^f \geq 0, \; \bar{x}_{ol} \in \{0,1\}, \; y_i \in \{0,1\}, \; i \in M, \; o, l \in O'. \tag{32}$$

Additional constraints

$$\bar{x}_{oo'} \le z_{oo'},\ o, o' \in O', \tag{33}$$

$$\sum_{o \in O'} z_{oo'} + \sum_{o \in O' \setminus \{0\}} z_{o'o} = nm + 1,\ o' \in O' \setminus \{0\}. \tag{34}$$

Lower Bound: Total Completion Time

$$\sum_{i=1}^{m}\left(\sum_{j=1}^{n} p_{\sigma_i,j}\right)(m-i+1) + m\sum_{j=1}^{n}\min\{\min_{j'\neq j} s_{j',j};\ \ s'_j\}$$

Experiment: Total Completion Time (50 cust, 25 prod)

Deviations from the record, %.

		Gurobi							
		Objective function				Lower bound			
Res.	LB0	M1	M2	M3	M4	M1	M2	M3	M4
min	0	0	8	2	12	95	62	91	94
max	0	5	49	40	46	100	97	93	94
aver	0	0	32	12	26	97	73	92	94

		SCIP							
		Objective function				Lower bound			
Res.	LB0	M1	M2	M3	M4	M1	M2	M3	M4
min	0	2	28	6	15	97	92	96	92
max	0	23	87	40	69	100	97	100	98
aver	0	14	47	22	40	98	95	98	94

Experiment: Weighted Throughput (50 cust, 25 prod)

Deviations from the record, %.

	Gurobi				SCIP			
Res.	M1	M2	M3	M4	M1	M2	M3	M4
min	0	0	0	0	0	15	13	15
max	42	64	53	190	53	96	65	1060
aver	10	27	23	81	24	62	49	332

Memetic Algorithm

Generate tentative solutions to form the initial population.

Repeat until the stopping criterion is met.

2.1: Select two solutions from the current population.

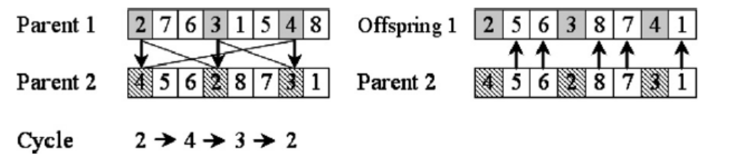
2.2: Build offspring by a recombination (crossover) and a mutation.

2.3: Choose solutions for the next generation.

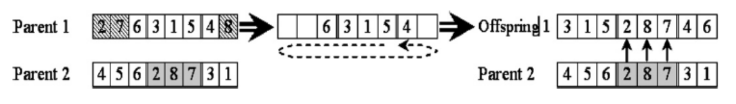
Return the best found solution.

Crossover Operators

Cycle Crossover (CX)

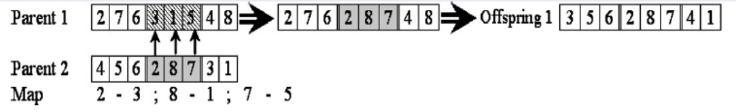


Order Crossover (OX)

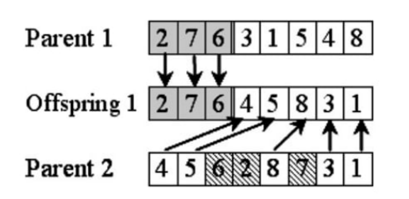


Crossover Operators

Partially Mapped Crossover (PMX)

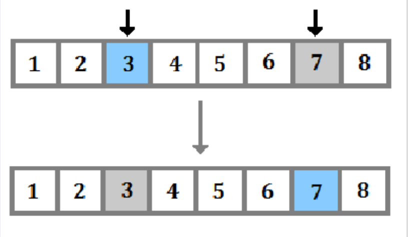


One Point Crossover (1PX)

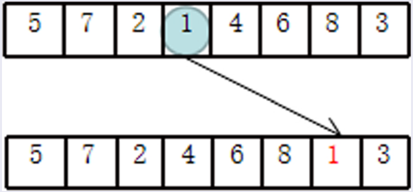


Mutation Operators

Exchange (swap) mutation



Shift (insert) mutation



Optimal Recombination

We have two parent permutations π^1 and π^2 . It is required to find a permutation π' such that:

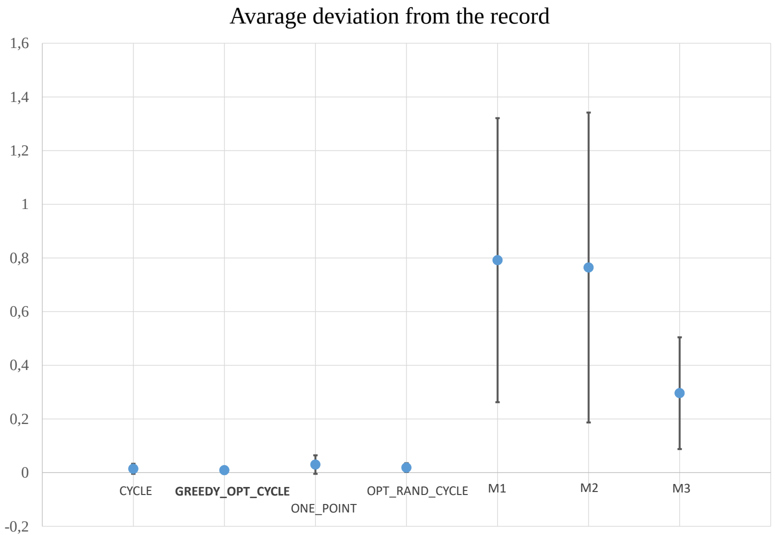
(I) $\pi'_i = \pi^1_i$ or $\pi'_i = \pi^2_i$ for all $i = 1, \ldots, nm$;

(II) π' has the optimum value of objective function among all permutations that satisfy condition (I).

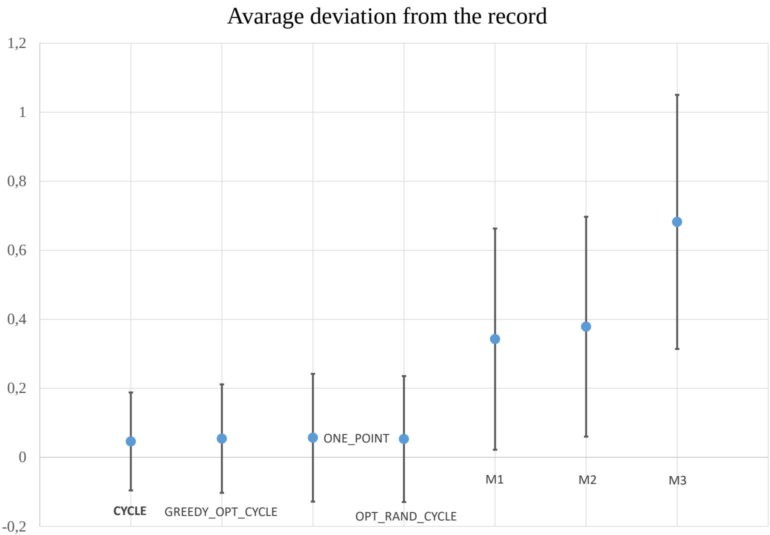
The optimal recombination problem for the weighted throughput criterion is NP-hard.

The complexity status of the ORP for the total completion time criterion is an open question.

Experimental Results: Total completion time

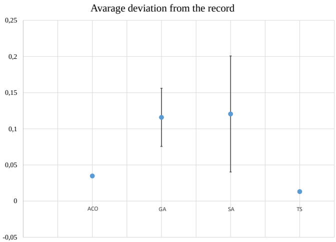


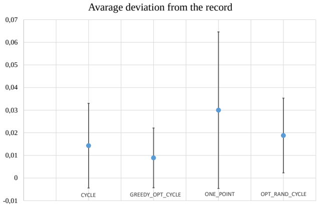
Experimental Results: Weighted Throughput



Experimental Results

Hazir, O., Gunalay, Y., Erel, E. (2008). Customer order scheduling problem: a comparative metaheuristics study:
$n:\ 5,\ 10,\ 15,\ 20,\ m:\ 5,\ 10,\ 15,\ 20$
setup times: $(10, 20),\ (1, 30),\ (25, 35),\ (15, 45)$
durations: $(1, 15)$





Conclusion

We provide and compare several approaches to construct integer programming models for the problem.

We propose a memetic algorithm for searching near optimal solutions.

The results of the experimental evaluation on instances of various structures are analysed.

Thank you for your attention!