Исследование множества Парето двухкритериальной задачи по выполнению многокомпонентных заказов клиентов

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Discrete Multicriteria Problem

Discrete Problem

X is the finite set of feasible solutions
$f = (f_1, f_2, \dots, f_m)$ is the vector criterion

The Pareto relation

$f(x^*) \geq f(x)$ means that $f(x^*) \neq f(x)$ and $f_i(x^*) \geqslant f_i(x)$ for all $i \in I = \{1, 2, \dots, m\}$

Pareto-optimal Solutions and The Pareto Set

$P_f(X) = \{x \in X \mid \nexists x^* \in X : f(x^*) \;\geq\; f(x)\;\}$
$P(Y) = f(P_f(X))$

We assume that the Pareto set is defined except for a collection of equivalence classes (equivalence relation $x' \sim x''$ iff $f(x') = f(x'')$).

Main Approach (V. Noghin)

Multicriteria choice problem $< X, f, \succ >$:

- a set of feasible solutions X;
- a vector criterion $f = (f_1, f_2, \ldots, f_m)$ defined on set X;
- an asymmetric binary preference relation of the DM $\succ$ defined on set $Y = f(X)$.

The preference relation of the DM

$f(x') \succ f(x'')$ means that the DM prefers the solution x' to x''.

Binary relation $\succ$ of the DM's preferences (axioms of "reasonable" choice)

1. irreflexive: $f(x) \succ f(x)$ is not true for any $x \in X$;
2. transitive: if $f(x') \succ f(x'')$, $f(x'') \succ f(x''')$, then $f(x') \succ f(x''')$ for any $x', x'', x'' \in X$;
3. invariant with respect to a linear positive transformation: $f(x') \succ f(x'') \quad \text{iff} \quad \alpha f(x') + y \succ \alpha f(x'') + y$ for any $x', x'' \in X$, $y \in \mathbb{R}^m$, $\alpha \in \mathbb{R}$, $\alpha > 0$
4. compatibility: $f(x') \succ f(x'')$, if $f_i(x') > f_i(x'')$, $f_s(x') = f_s(x'')$ for any $s \in I$, $s \neq i$ for any $i \in I$, $x', x'' \in X$
5. if $f(x') \succ f(x'')$ for some $x', x'' \in X$, then the solution x'' could not be chosen from the whole set X

Edgeworth–Pareto principle

Set of selected outcomes $Ch(Y)$ is interpreted as some abstract set corresponded to the set of solutions, that satisfy all hypothetic preferences of the DM.
Thus, set $Ch(Y)$ is considered as **the optimal choice of particular DM**.

Edgeworth–Pareto principle

Under axioms of "reasonable" choice for any set of selected outcomes $Ch(Y)$

$$Ch(Y) \subseteq P(Y)^{a} \quad (1)$$

[a] Noghin, V.D.: Reduction of the Pareto Set: An Axiomatic Approach. Springer International Publishing (2018)

The idea of the Pareto set reduction approach is to construct a narrower upper bound (than (1)) on the optimal choice $Ch(Y)$ using additional information about the DM's preference relation $\succ$.

"Quantum of information"

Definition 1

We say that there exists a *"quantum of information"* about the DM's preference relation $\succ$ if the vector $y \in \mathbb{R}^m$ such that

$$y_i = 1 - \theta > 0, \quad y_j = -\theta < 0, \quad y_s = 0 \quad \forall s \in I \setminus \{i, j\} \qquad (2)$$

satisfies the expression $y \succ 0_m$.[a]

In such case we will say, that component of criteria i is more important than component j with coefficient of compromise θ. Here, $\theta \in (0, 1)$.

[a] Noghin, V.D.: Reduction of the Pareto Set: An Axiomatic Approach. Springer International Publishing (2018)

The coefficient of compromise θ shows the quantity of relative loss.

Reduction of the Pareto set

Theorem 1 (Noghin)

Given a "quantum of information" by Definition 1, the inclusions

$$Ch(Y) \subseteq \hat{P}(Y) \subseteq P(Y)$$

are valid for any set of selected outcomes $Ch(Y)$. Here $\hat{P}(Y) = f(P_{\hat{f}}(X))$, and $P_{\hat{f}}(X)$ is the set of pareto-optimal solutions with respect to m-dimensional vector criterion $\hat{f} = (\hat{f}_1, \dots, \hat{f}_m)$:

$$\hat{f}_j = \theta f_i + (1-\theta) f_j, \quad \hat{f}_s = f_s \quad \forall s \neq j.^{a} \tag{3}$$

[a] Noghin, V.D.: Reduction of the Pareto Set: An Axiomatic Approach. Springer International Publishing (2018)

Bi-criteria Scheduling Problem

Problem Statement

n is the number of products
p_j is the processing time of job j
$s_{jj'}$ is the setup time between products j and j'
s'_j is the initial setup time for product j
d_j is the due date for product j
w_i is the weight of product j

Criteria

Then the first criterion is $f_1(\pi) = -\sum_{i=1}^{n} C_i$.

The second criterion is $f_2(\pi) = \sum_{i:\ C_i \le d_i} w_i$.

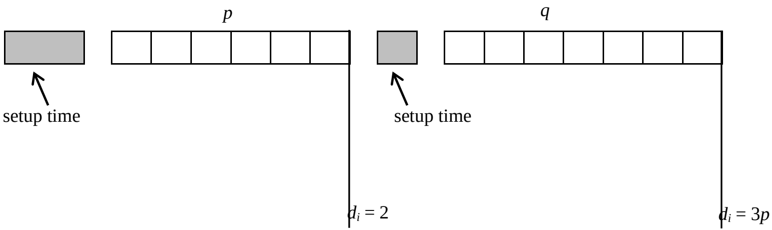
Series of Instances

Input Data

p products with unit processing times and q products with processing times equal to p.

The initial setup times:
$s'_i = \frac{i}{p+q}$ for $i = 1, \ldots, p$ and $s'_i = +\infty$ for $i = p+1, \ldots, p+q$.
The setup times between products:
$s_{ii'} = 0$ for $i, i' = 1, \ldots, p$;
$s_{ii'} = \frac{(i'-1-p)p}{q}$ and $s_{i'i} = +\infty$ for $i = 1, \ldots, p$ and
$i' = p+1, \ldots, p+q$;
$s_{ii'} = 0$ for $i, i' = p+1, \ldots, p+q$.

Products $i = 1, \ldots, p$ have a common due date 2 and weights $w_i = i$.
Products $i = p+1, \ldots, p+q$ have a common due date $3p$ and weights $w_i = C + (i-p-1)(2p-1)$.


p
q
setup time
setup time
$d_i = 2$
$d_i = 3p$

$f_1 \longrightarrow f_2$ with coefficient of compromise θ:

- if $\theta \in (0, \frac{1}{2})$, then the reduction does not hold;
- if $\theta \in [\frac{1}{2}, \frac{2p-1}{3p-1})$, then the reduced Pareto set consists of q elements;
- if $\theta \in [\frac{2p-1}{3p-1}, 1)$, then the reduced Pareto set consists of one element.

$f_2 \longrightarrow f_1$ with coefficient of compromise θ:

- if $\theta \in (0, \frac{p}{3p-1})$, then the reduced Pareto set contains at least $p + q - 1$ elements;
- if $\theta \in [\frac{p}{3p-1}, \frac{1}{2})$, the reduced Pareto set consists of p elements;
- if $\theta \in [\frac{1}{2}, 1)$, then the reduced Pareto set consists of one element.

Approximation of the Pareto Set

- Find an optimal solution of the single objective problem with the first criterion. Let $f_1^* = (f_{11}^*, f_{12}^*)$ denote the objective values of the optimal solution. Put the current iteration number $iter = 1$.
- Repeat until $iter \leq m_{\max}$.
  2.1 Put $iter = iter + 1$.
  2.2 Find optimal solution of single objective problem with additional constraint $f_1 \geq f_{1,iter-1}^* + \delta$.
  2.3 Let $f_{iter}^* = (f_{iter,1}^*, f_{iter,2}^*)$ denote the obtained optimal objective values.
- Find an approximation of the Pareto set from $\{f_1^*, \ldots, f_{m_{\max}}^*\}$.

Computational Experiment

Instance	|Approximation| 1	3	5
Inst1	16	13	13
Inst2	13	13	13
Inst3	14	14	13
Inst4	12	12	11
Inst5	13	13	13
Inst6	18	18	18
Inst7	17	17	17
Inst8	14	14	14
Inst9	16	15	12
Inst10	11	11	11

Thank you for your attention!