Investigation of operators and parameters in evolutionary algorithms for one scheduling problem with resource constraints

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Speed Scaling Scheduling

Processors and Jobs

2 speed-scalable processors

$\mathcal{J} = \{1, \ldots, n\}$ is the set of jobs:
V_j is the processing volume (work) of job j
$size_j$ is the number of processors required by job j
$W_j := \frac{V_j}{size_j}$ is the work on one processor
E is the energy budget

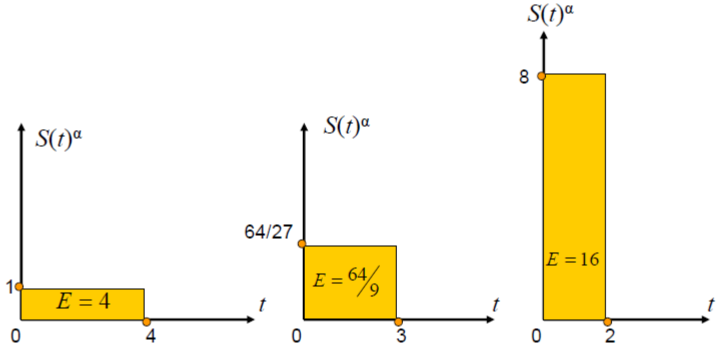
Parameters

Preemption and migration are characterized for the systems with single image of the memory.
Non-preemptive instances arise in systems with distributed memory.

Homogeneous Model in Speed-scaling

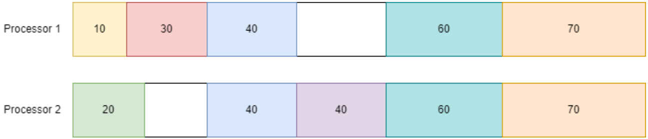
If a processor runs at speed s then the energy consumption is s^α units of energy per time unit, where $\alpha > 1$ is a constant (practical studies show that $\alpha \leq 3$).

It is supposed that a continuous spectrum of processor speeds is available.



The aim is to find a feasible schedule with the minimum total completion time so that the energy consumption is not greater than a given energy budget.

Solution



Lower Bound

Previous Research

Scheduling

- Lee & Cai: Scheduling one and two-processor tasks on two parallel processors (1999)
- Kononov & Zakharova: Speed scaling scheduling of multiprocessor jobs with energy constraint and total completion time criterion (2023)
- Zakharova & Sakhno: Heuristics with local improvements for two-processor scheduling problem with energy constraint and parallelization (2024)

Evolutionary Computation

- Eremeev & Kovalenko: A memetic algorithm with optimal recombination for the asymmetric travelling salesman problem (2020)
- Neri & Cotta: Memetic Algorithms and Memetic Computing Optimization: A Literature Review (2012)
- Blum & Eremeev & Zakharova: Hybridizations of evolutionary algorithms with Large Neighborhood Search (2022)
- Doerr & Ghannane, & Ibn Brahim: Runtime Analysis for Permutation-based Evolutionary Algorithms (2024)

Genetic Algorithm (GA) with Generational Scheme

1: Construct the initial population $P^0 = \{\pi_j^0\}$ of k permutations. Save n_e individuals with the best objective values as elites of P^0. Put $t = 0$.

2: Until termination condition is met, perform

2.1 for $i \leftarrow 1$ to $(k - n_e)/2$

2.1.1 Select two parent permutations π^1 and π^2 using operator $Sel(P^t)$.

2.1.2 Construct $(\pi^{1\prime}, \pi^{2\prime}) = Cross(\pi^1, \pi^2)$.

2.1.3 Apply the mutation operator to constructed permutations: $Mut(\pi^{1\prime})$ and $Mut(\pi^{2\prime})$ and save the result as individuals $\pi_{2i-1}^{t+1}, \pi_{2i}^{t+1}$ for population P^{t+1}.

2.2 Copy elites of P^t to P^{t+1} and identify elites of P^{t+1}.

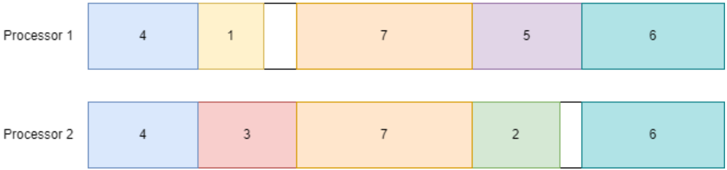
2.3 Put $t = t + 1$.

3: Return the best found individual.

Solution encoding

The solutions are encoded by permutations of jobs.

4, 1, 3, 7, 5, 2, 6



Crossover Operators

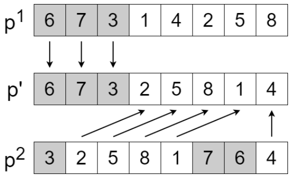


Figure: One Point Crossover (1PX)

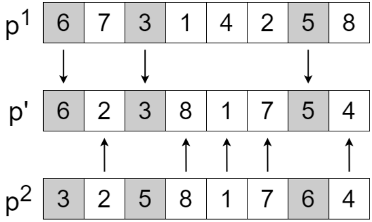


Figure: Cycle Crossover (CX)

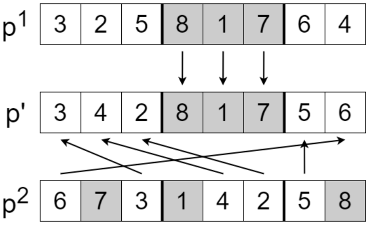


Figure: Order Crossover (OX)

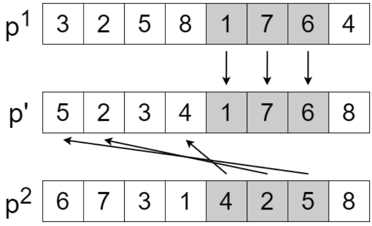
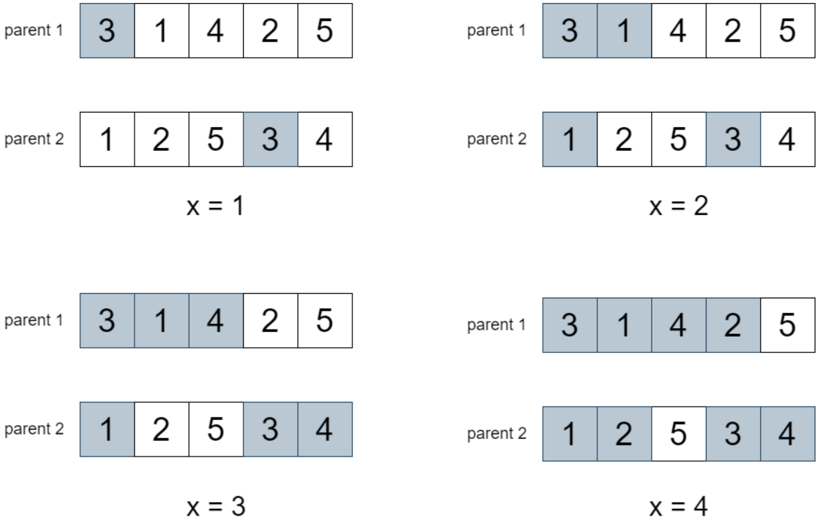


Figure: Partially Mapped Crossover (PMX)

Optimized Crossovers

One Point Crossover (1PX)



Mutation Operators

Exchange (swap) mutation

3	8	4	1	5	2	7	6	⟹	3	8	7	1	5	2	4	6

Shift (insert) mutation

3	8	4	1	5	2	7	6	⟹	3	8	4	5	2	7	1	6

Scramble Mutation Scheme[1]

1. Randomly choose n_p from Poisson distribution with λ_p.
2. Apply operator Mut for the given genotype n_p times.

[1] Doerr & Ghannane, & Ibn Brahim: Runtime Analysis for Permutation-based Evolutionary Algorithms (2024)

Adaptive Technique[2]

1: Choose a crossover. The probability of choosing each operator is proportional to its weight.
2: Apply chosen crossover to the parent genotypes.
3: Update the weight of the chosen crossover:

$$\phi_a = \begin{cases} w_1, \text{if the new solution is a new global best,} \\ w_2, \text{if the new solution is better than the current one,} \\ w_3, \text{if the new solution is better than one of the parents or both.} \end{cases}$$

$$\rho_a = \lambda \rho_a + (1 - \lambda)\phi_a.$$

[2] Mara & Norcahyo & Jodiawan & Lusiantoro & Rifai: A survey of adaptive large neighborhood search algorithms and applications (2022)

Parameter auto-tuning: IRACE package[3]

Parameter name	Parameter description
k	population size
n_e	number of elites
P_{IPRand}	probability of generating a genotype randomlly
$Selection$	selection operator
P_{Cross}	probability of applying the crossover operator
$Crossover$	crossover operator
P_{Mut}	probability of applying the mutation operator
$Mutation$	mutation operator
w_2, w_3, λ	parameters of adaptive technique
λ_p	lambda for Poisson distribution

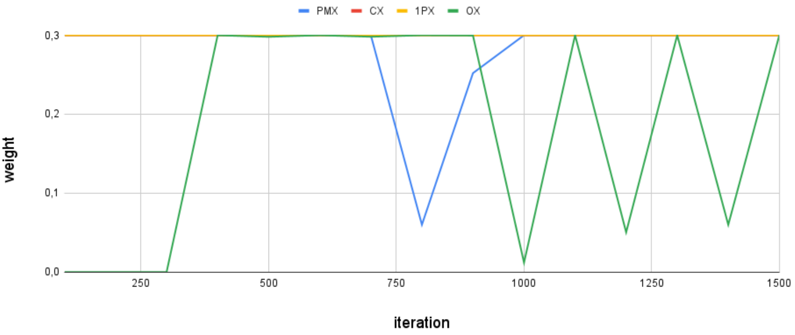
[3] Lopez-Ibanez, M., Dubois-Lacoste, J., Perez Caceres, L, Birattari, M., Stutzle, T.: The irace package: Iterated racing for automatic algorithm configuration, Operations Research Perspectives, 3, 43-58 (2016)

Versions of genetic algorithm

GA_{rand} is a GA.

GA_{adapt_rand} is the GA with the adaptive technique for randomized crossover operators ($1PX$, CX, OX, PMX).

Dynamics of crossover weights during GA_{adapt_rand} iterations



The classic restarting rule is used.

Versions of genetic algorithm

GA_{rand} is a classic GA.

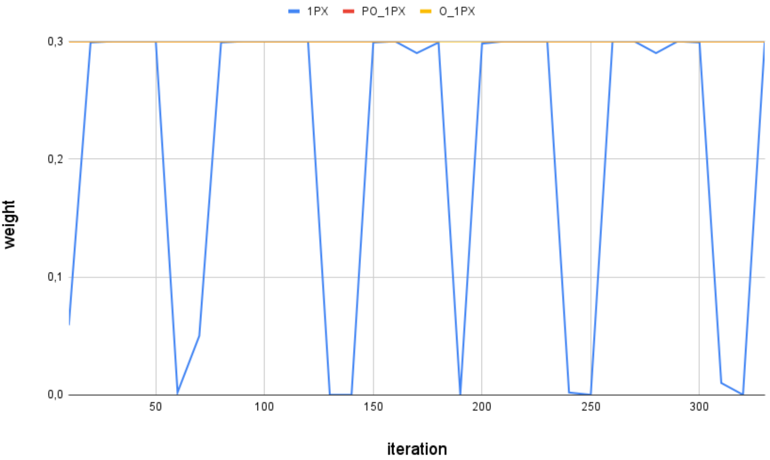
GA_{adapt_rand} is the GA with the adaptive technique for randomized crossover operators ($1PX$, CX, OX, PMX).

GA_{adapt_opt} is the GA with the adaptive technique for optimized crossover operators (1PX, PO_1PX, O_1PX).

GR_{LI} is the known greedy heuristic with local improvements[4].

[4] Zakharova & Sakhno: Heuristics with local improvements for two-processor scheduling problem with energy constraint and parallelization (2024)

Dynamics of crossover weights during GA_{adapt_opt} iterations



The classic restarting rule is used.

Experiment result

	GA_{rand}	GA_{adapt_rand}	GA_{adapt_opt}	GR_{LI}
avg	1.99%	2.05%	1.94%	4.56%
min	0.82%	0.83%	0.81%	1.67%
max	3.86%	3.76%	3.63%	7.74%

Table: Relative deviations of results from the lower bound for algorithms with parameters found by IRACE package

	$GA_{rand\_poisson}$	$GA_{adapt\_rand\_poisson}$	$GA_{adapt\_opt\_poisson}$
avg	1.96%	1.96%	1.95%
min	0.83%	0.86%	0.8%
max	3.72%	3.57%	3.63%

Table: Relative deviations of results from the lower bound for algorithms with scramble mutation operator and with parameters found by IRACE package

Conclusions and Further Research

Recommendations

- Apply auto-tuning for parameters of algorithm.
- Apply adaptive technique to identify the leading crossover operator.
- Implement optimized version of the leading crossover operator and try to apply scramble mutation.

Further Plans

- Generalize the algorithm on permutation problems.
- Compare with other known algorithms
  (P.A. Borisovsky, "A parallel "Go with the winners" algorithm for some scheduling problems", 2023;
  P. Borisovsky, Y. Kovalenko, "A Memetic Algorithm with Parallel Local Search for Flowshop Scheduling Problems", 2020).

Thank you for your attention!

Convex program

$$\sum_{j\in\mathcal{J}} C_j(\pi) = \sum_{j=1}^{n}(n-j+1)p_{\pi_j} \to \min, \qquad (1)$$

$$\sum_{j\in\mathcal{J}} (2p_j)^{1-\alpha}(V_j)^{\alpha} = E. \qquad (2)$$

Experiment result

	GA_{rand}	GA_{adapt_rand}	GA_{adapt_opt}	GR_{LI}
avg	2.03%	2.06%	1.95%	4.56%
min	0.83%	0.83%	0.78%	1.67%
max	3.83%	3.88%	3.57%	7.74%

Table: Relative deviations of results from the lower bound for algorithms

	GA_{rand}	GA_{adapt_rand}	GA_{adapt_opt}	GR_{LI}
avg	1.99%	2.05%	1.94%	4.56%
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