Instance reduction for the routing open shop problem and the problem generalization with extended jobs

Ilya Chernykh

Sobolev Institute of Mathematics, Novosibirsk

85th Workshop on Algorithms and Complexity
19.09.2023

The research was supported by
Russian Science Foundation grant N 22-71-10015

Open Shop ($Om||C_{max}$)...

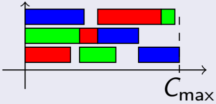
Machines M_1 $\cdots$ M_m

Jobs J_1 $\cdots$ J_n

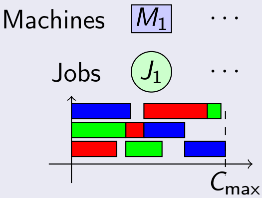
Open Shop ($Om||C_{\max}$)...

Machines M_1 $\cdots$ M_m

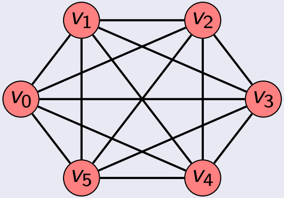
Jobs J_1 $\cdots$ J_n



Open Shop ($Om||C_{max}$)...



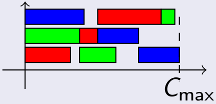
Metric TSP...



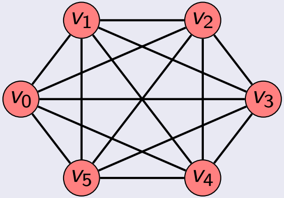
Open Shop ($Om||C_{\max}$)...

Machines M_1 $\cdots$ M_m

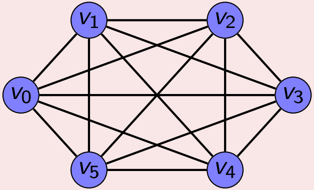
Jobs J_1 $\cdots$ J_n



Metric TSP...



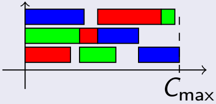
... and their combination $ROm||R_{\max}$



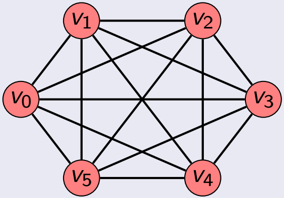
Open Shop ($Om||C_{max}$)...

Machines M_1 $\cdots$ M_m

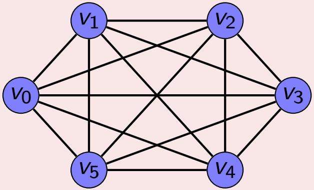
Jobs J_1 $\cdots$ J_n



Metric TSP...



... and their combination $ROm||R_{max}$

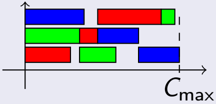


$\{J_1, \ldots, J_n\}$

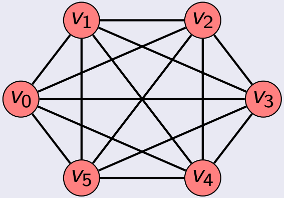
Open Shop ($Om||C_{max}$)...

Machines M_1 $\cdots$ M_m

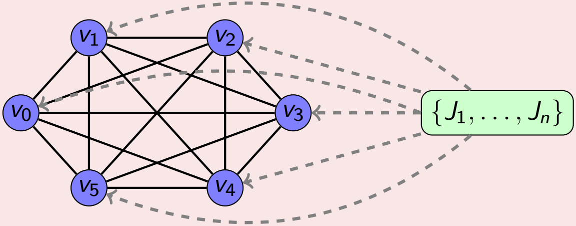
Jobs J_1 $\cdots$ J_n



Metric TSP...



... and their combination $ROm||R_{max}$

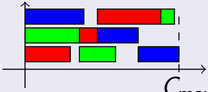


Routing open shop
informal intro

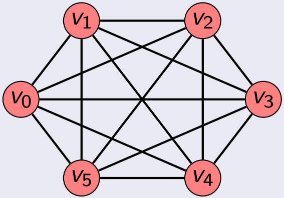
Open Shop ($Om||C_{max}$)...

Machines M_1 $\cdots$ M_m

Jobs J_1 $\cdots$ J_n



Metric TSP...

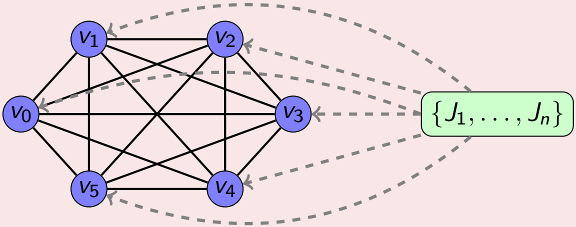


... and their combination $ROm||R_{max}$

M_1

$\vdots$

M_m

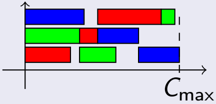


Routing open shop
informal intro

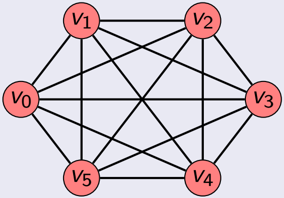
Open Shop ($Om||C_{max}$)...

Machines M_1 $\cdots$ M_m

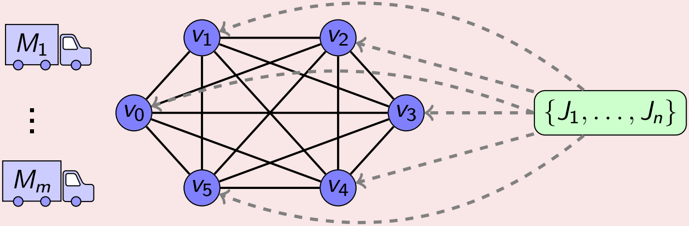
Jobs J_1 $\cdots$ J_n



Metric TSP...



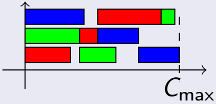
... and their combination $ROm||R_{max}$



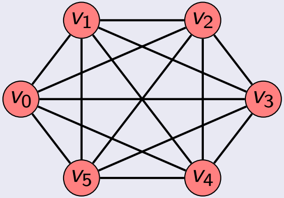
Open Shop ($Om||C_{max}$)...

Machines M_1 $\cdots$ M_m

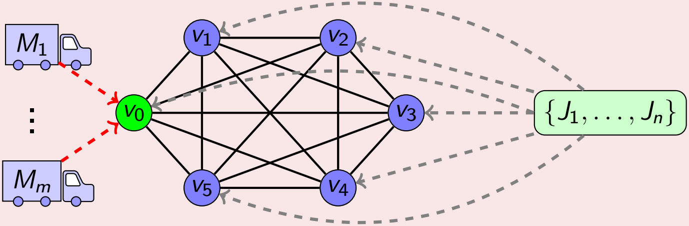
Jobs J_1 $\cdots$ J_n



Metric TSP...



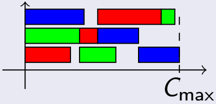
... and their combination $ROm||R_{max}$



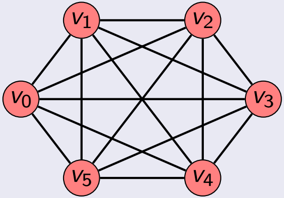
Open Shop ($Om||C_{\max}$)...

Machines M_1 $\cdots$ M_m

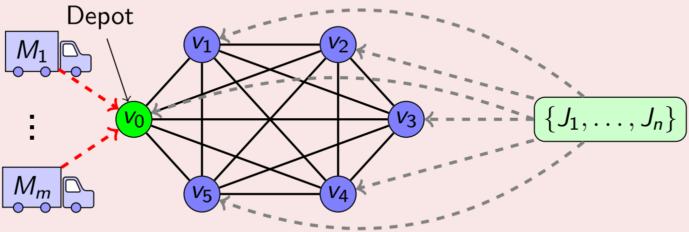
Jobs J_1 $\cdots$ J_n



Metric TSP...



... and their combination $ROm||R_{\max}$



Complexity review

$RO1||R_{\max}$

Equivalent to metric TSP; strongly NP-hard

$ROm|G = K_1|R_{\max}$

Equivalent to $Om||C_{\max}$:

1. Solvable in linear time for $m = 2$ [Gonzalez, Sahni 1976]
2. NP-hard for $m \geqslant 3$ [Gonzalez, Sahni 1976]; PTAS exists for any $m = \mathrm{const}$ [Sevastyanov, Woeginger 1998]
3. Strongly NP-hard for unbounded m; cannot be approximated with ratio better than $5/4$ [Williamson *et al* 1997]

$RO2|G = K_2|R_{\max}$

1. NP-hard [Averbakh, Berman, Ch 2006]
2. Also NP-hard in proportionate case [Pyatkin, Ch 2022]
3. FPTAS exists [Kononov 2012]

Standard lower bound

- p_{ji} — processing time;
- $\ell_i = \sum_{j=1}^{n} p_{ji}$ — machine load of M_i, $d_j = \sum_{i=1}^{m} p_{ji}$ — job length of J_j,
- $\ell_{\max} = \max \ell_i$ — maximum machine load,
- $d_{\max}(v) = \max_{J_j \in \mathcal{J}(v)} d_j$ — maximum length of job from v,
- $G = \langle V, E \rangle$ — the transportation network,
- T^* — TSP optimum on G,
- $v_0 \in V$ — the depot,
- $\mathrm{dist}(u, v)$ — travel time between u и v.

Standard lower bound

- p_{ji} — processing time;
- $\ell_i = \sum_{j=1}^{n} p_{ji}$ — machine load of M_i, $d_j = \sum_{i=1}^{m} p_{ji}$ — job length of J_j,
- $\ell_{\max} = \max \ell_i$ — maximum machine load,
- $d_{\max}(v) = \max_{J_j \in \mathcal{J}(v)} d_j$ — maximum length of job from v,
- $G = \langle V, E \rangle$ — the transportation network,
- T^* — TSP optimum on G,
- $v_0 \in V$ — the depot,
- $\mathrm{dist}(u, v)$ — travel time between u и v.

Standard lower bound for $ROm||R_{\max}$

$$\bar{R} = \max\left\{\ell_{\max} + T^*, \max_{v \in V}\left(d_{\max}(v) + 2\mathrm{dist}(v_0, v)\right)\right\}$$

Optima localization review

Single node (open shop)

- $RO2|G = K_1|R_{\max}$: $OPT = \bar{R}$ [Gonzalez, Sahni 1976]
- $RO3|G = K_1|R_{\max}$: $OPT \in [\bar{R}, \frac{4}{3}\bar{R}]$ [Sevastyanov, Ch 1998]

Optima localization review

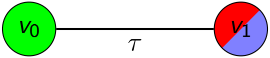
Single node (open shop)

- $RO2|G = K_1|R_{\max}$: $OPT = \bar{R}$ [Gonzalez, Sahni 1976]
- $RO3|G = K_1|R_{\max}$: $OPT \in [\bar{R}, \frac{4}{3}\bar{R}]$ [Sevastyanov, Ch 1998]

More than one node

- $RO2|G = K_2|R_{\max}$: $OPT \in [\bar{R}, \frac{6}{5}\bar{R}]$ [Averbakh, Berman, Ch 2005]
- $RO2|G = K_3|R_{\max}$: $OPT \in [\bar{R}, \frac{6}{5}\bar{R}]$ [Lgotina, Ch 2016]
- $RO2|G = tree|R_{\max}$: $OPT \in [\bar{R}, \frac{6}{5}\bar{R}]$ [Krivonogova, Ch 2019]
- $RO3|G = K_2|R_{\max}$: $OPT \in [\bar{R}, \frac{4}{3}\bar{R}]$ [Krivonogova, Ch 2020]

A critical instance with $OPT = 6/5\bar{R}$

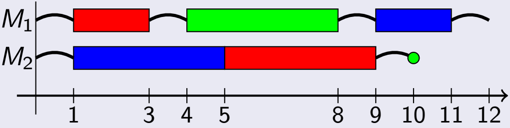


Standard lower bound

$$\bar{R} = \max\{\ell_{\max} + 2\tau, d_{\max}(v_0), d_{\max}(v_1) + 2\tau\}.$$

The instance

$$J_0 = (4,0);\ \tau = 1;\ J_1 = (2,4);\ J_2 = (2,4).$$



What's next?

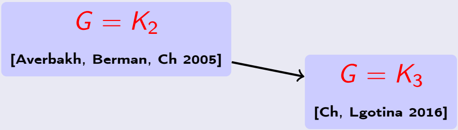
$RO2||R_{max}$: Is $OPT > \frac{6}{5}\bar{R}$ for some instance?

$G = K_2$

[Averbakh, Berman, Ch 2005]

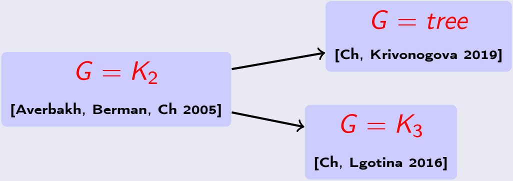
What's next?

$RO2||R_{max}$: Is $OPT > \frac{6}{5}\bar{R}$ for some instance?



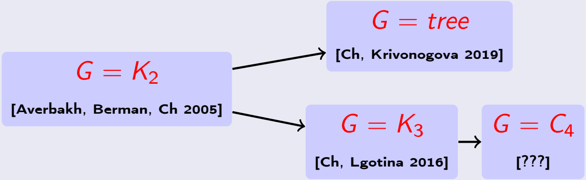
What's next?

$RO2||R_{\max}$: Is $OPT > \frac{6}{5}\bar{R}$ for some instance?



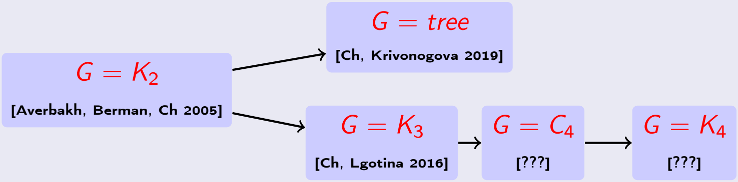
What's next?

$RO2||R_{max}$: Is $OPT > \frac{6}{5}\bar{R}$ for some instance?



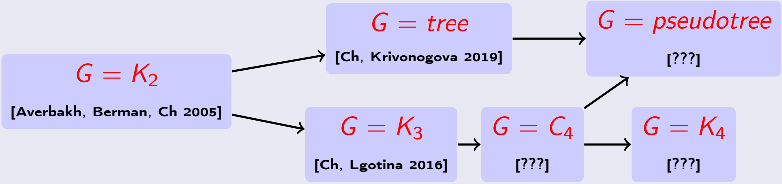
What's next?

$RO2||R_{\max}$: Is $OPT > \frac{6}{5}\bar{R}$ for some instance?

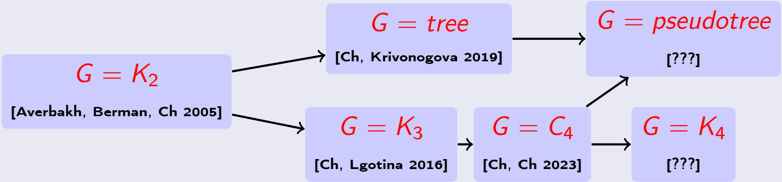


What's next?

$RO2||R_{max}$: Is $OPT > \frac{6}{5}\bar{R}$ for some instance?

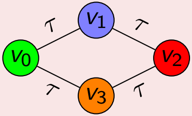


$RO2||R_{\max}$: Is $OPT > \frac{6}{5}\bar{R}$ for some instance?

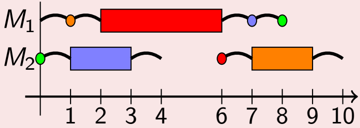


As a matter of fact, YES!

- $RO2|G = C_4|R_{\max}$: $OPT \in [\bar{R}, \frac{5}{4}\bar{R}]$ [Chechushkov, Ch 2023]



$\tau = 1$; $J_0 = (0, 0)$; $J_1 = (0, 2)$;
$J_2 = (4, 0)$; $J_3 = (0, 2)$.



What's next?

$RO2||R_{\max}$: Is $OPT > \frac{5}{4}\bar{R}$ for some instance?

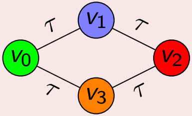
$G = K_2$ [Averbakh, Berman, Ch 2005] → $G = tree$ [Ch, Krivonogova 2019] → $G = pseudotree$ [???]

$G = K_2$ [Averbakh, Berman, Ch 2005] → $G = K_3$ [Ch, Lgotina 2016] → $G = C_4$ [Ch, Ch 2023] → $G = pseudotree$ [???]

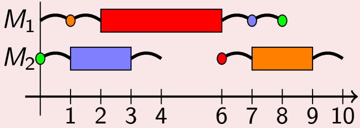
$G = C_4$ [Ch, Ch 2023] → $G = K_4$ [???]

As a matter of fact, YES!

- $RO2|G = C_4|R_{\max}$: $OPT \in [\bar{R}, \frac{5}{4}\bar{R}]$ [Chechushkov, Ch 2023]



$\tau = 1$; $J_0 = (0,0)$; $J_1 = (0,2)$;
$J_2 = (4,0)$; $J_3 = (0,2)$.



Instance reduction

General idea:

I

Instance reduction

General idea:

- Reduce given instance I to simpler $\tilde{I}$, preserving $\bar{R}$.

$$I \longrightarrow \tilde{I}$$

Instance reduction

General idea:

- Reduce given instance I to simpler $\tilde{I}$, preserving $\bar{R}$.
- Use some algorithm A to find a "good enough" solution $\tilde{S}$ for $\tilde{I}$.

$$\begin{array}{ccc} I & \longrightarrow & \tilde{I} \\ & & \Big\downarrow \mathcal{A} \\ & & \tilde{S} \end{array}$$

General idea:

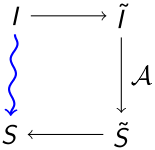
- Reduce given instance I to simpler $\tilde{I}$, preserving $\bar{R}$.
- Use some algorithm A to find a "good enough" solution $\tilde{S}$ for $\tilde{I}$.
- Interpret it as a feasible solution S for I with the some objective.

$$\begin{array}{ccc} I & \longrightarrow & \tilde{I} \\ & & \big\downarrow \mathcal{A} \\ S & \longleftarrow & \tilde{S} \end{array}$$

Instance reduction

General idea:

- Reduce given instance I to simpler $\tilde{I}$, preserving $\bar{R}$.
- Use some algorithm A to find a "good enough" solution $\tilde{S}$ for $\tilde{I}$.
- Interpret it as a feasible solution S for I with the some objective.



Known transformations. Job aggregation

Definition

Let $\mathcal{K} \subseteq \mathcal{J}(v)$ — some subset of jobs from v.
The following transformation is called aggregation operation of set $\mathcal{K}$:

$$\mathcal{J}'(v) = \mathcal{J}(v) \setminus \mathcal{K} \cup \{J_{\mathcal{K}}\},\ p_{\mathcal{K}i} = \sum_{J_j \in \mathcal{K}} p_{ji}.$$

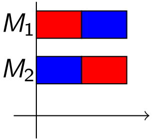
$J_{\mathcal{K}}$ is a new job to substitute $\mathcal{K}$.

Definition

Let $\mathcal{K} \subseteq \mathcal{J}(v)$ — some subset of jobs from v.
The following transformation is called aggregation operation of set $\mathcal{K}$:

$$\mathcal{J}'(v) = \mathcal{J}(v) \setminus \mathcal{K} \cup \{J_{\mathcal{K}}\}, \; p_{\mathcal{K}i} = \sum_{J_j \in \mathcal{K}} p_{ji}.$$

$J_{\mathcal{K}}$ is a new job to substitute $\mathcal{K}$.



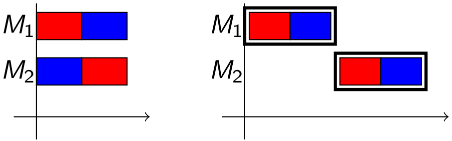
Known transformations. Job aggregation

Definition

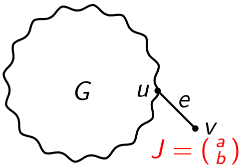
Let $\mathcal{K} \subseteq \mathcal{J}(v)$ — some subset of jobs from v.
The following transformation is called aggregation operation of set $\mathcal{K}$:

$$\mathcal{J}'(v) = \mathcal{J}(v) \setminus \mathcal{K} \cup \{J_{\mathcal{K}}\}, \; p_{\mathcal{K}i} = \sum_{J_j \in \mathcal{K}} p_{ji}.$$

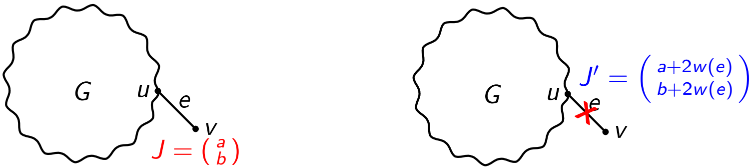
$J_{\mathcal{K}}$ is a new job to substitute $\mathcal{K}$.



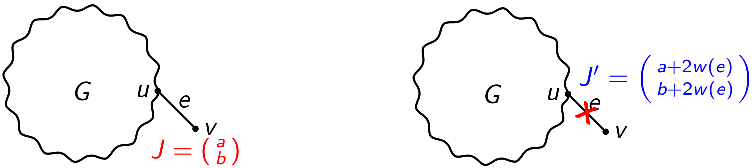
Terminal edge contraction

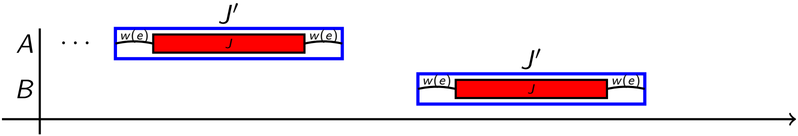


Terminal edge contraction

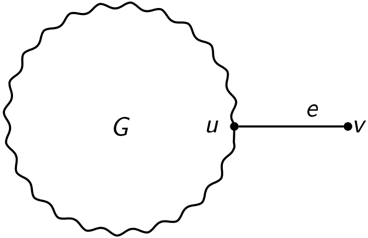


Terminal edge contraction

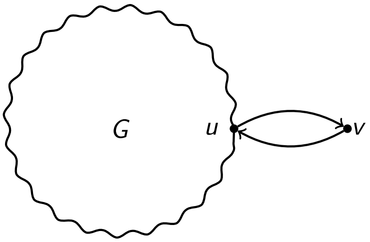




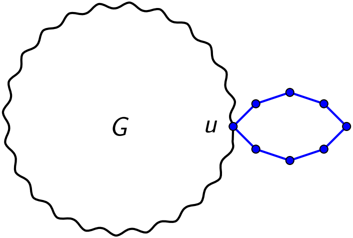
Terminal cycle contraction



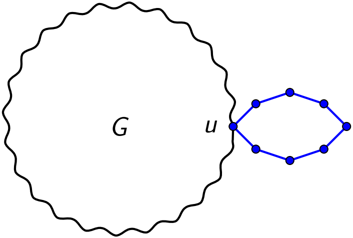
Terminal cycle contraction



Terminal cycle contraction



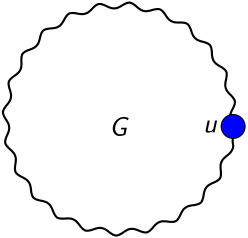
Terminal cycle contraction



Definition

Cycle is terminal if it contains the only *gate* (node of degree > 2 or depot v_0).

Terminal cycle contraction



Definition

Cycle is terminal if it contains the only *gate* (node of degree > 2 or depot v_0).

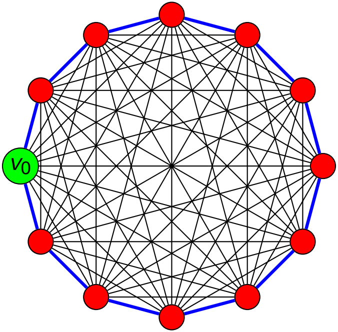
Lemma 1

It is possible to reduce the number of jobs by means of job aggregation in such way, that $\bar{R}$ is preserved AND there are at most $m-1$ nodes with more that 1 job each, containing in total at most $2m-1$ jobs.

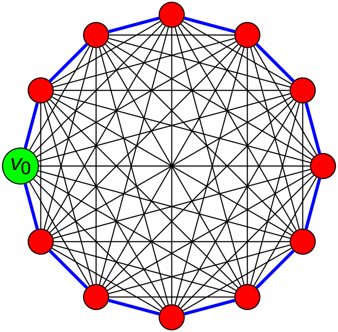
Lemma 2

It is possible to eliminate all but at most $m-1$ terminal elements by means of edge and cycle contraction in such way, that $\bar{R}$ is preserved.

Further instance reduction techniques



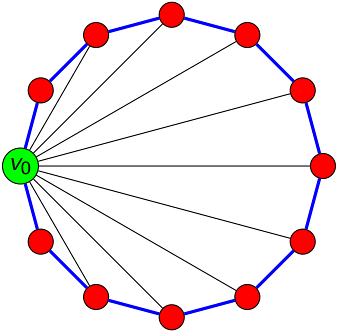
Further instance reduction techniques



Definition

Edges not belonging the optimal route are chords. Chords incident to the depot are radii. Chord is critical if its removal leads to increasing of $\bar{R}$.

Further instance reduction techniques



Definition

Edges not belonging the optimal route are chords. Chords incident to the depot are radii. Chord is critical if its removal leads to increasing of $\bar{R}$.

Properties of critical chords

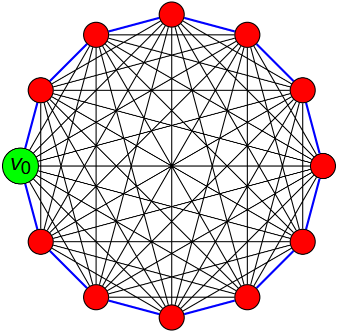
Lemma 3

Let I be an instance of $ROm||R_{\max}$, G is complete and with metric distances. When removal of all the chords except for the radii doesn't increase the value of $\bar{R}$.

Lemma 4

Let instance I of $ROm||R_{\max}$ contain $m-1$ critical radii. Then removal of all the other chords doesn't increase the value of $\bar{R}$.

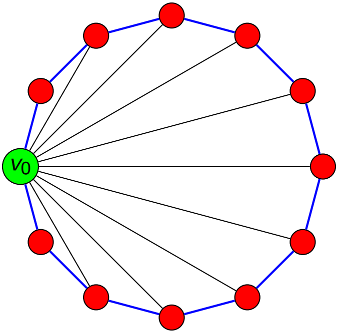
How does it work for $m = 2$?



Obsolete chords removal

1. Add radii to G, if necessary.

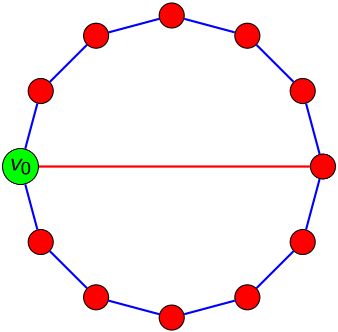
How does it work for $m = 2$?



Obsolete chords removal

1. Add radii to G, if necessary.
2. Remove every non-radius chord.

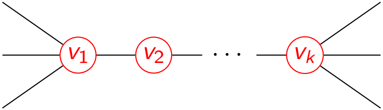
How does it work for $m = 2$?



Obsolete chords removal

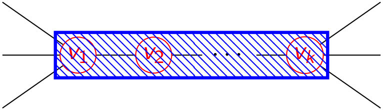
1. Add radii to G, if necessary.
2. Remove every non-radius chord.
3. Remove all the radii with possible exception of single critical.

Chain contraction: an idea



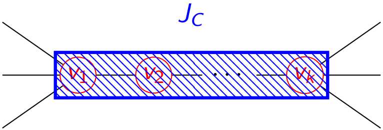
Consider a chain $C = (v_1, v_2, \ldots, v_k)$ such that all nodes $v_2, \ldots, v_{k-1}$ are of degree 2.

Chain contraction: an idea

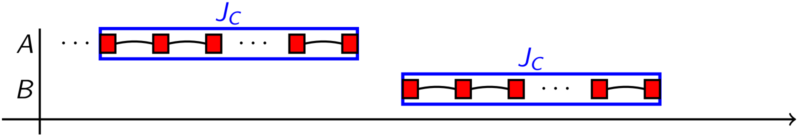


Consider a chain $C = (v_1, v_2, \ldots, v_k)$ such that all nodes $v_2, \ldots, v_{k-1}$ are of degree 2.

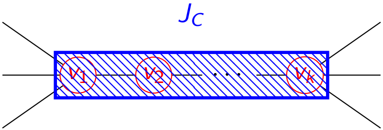
Chain contraction: an idea



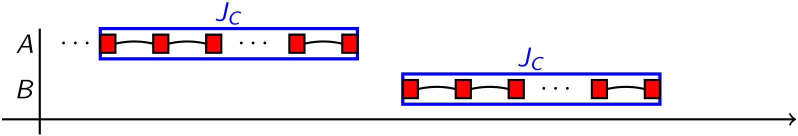
Consider a chain $C = (v_1, v_2, \ldots, v_k)$ such that all nodes $v_2, \ldots, v_{k-1}$ are of degree 2.



Chain contraction: an idea



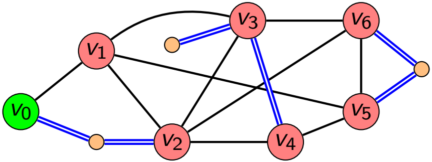
Consider a chain $C = (v_1, v_2, \ldots, v_k)$ such that all nodes $v_2, \ldots, v_{k-1}$ are of degree 2.



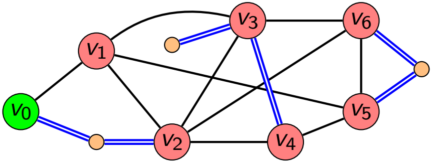
Definition

New edges obtained by chain contractions (and containing jobs) will be referred to as tunnels.

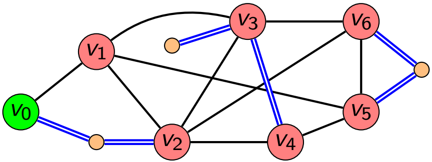
A routing open shop problem with tunnels



A routing open shop problem with tunnels

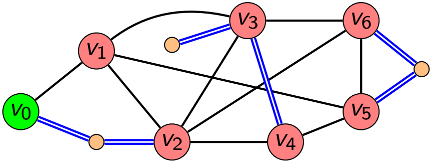


- v_0 is the depot.



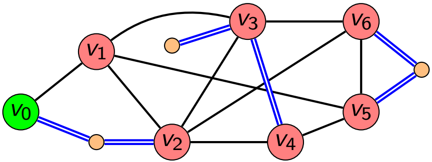
- v_0 is the depot.
- Some nodes (red ones plus depot) contain jobs (perhaps multiple jobs per node). Orange nodes do not contain jobs.

A routing open shop problem with tunnels



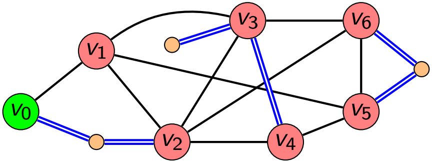
- v_0 is the depot.
- Some nodes (red ones plus depot) contain jobs (perhaps multiple jobs per node). Orange nodes do not contain jobs.
- Some edges (black) are used for traversing (as usual, it takes a time to travel over an edge).

A routing open shop problem with tunnels



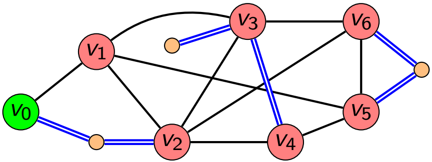
- v_0 is the depot.
- Some nodes (red ones plus depot) contain jobs (perhaps multiple jobs per node). Orange nodes do not contain jobs.
- Some edges (black) are used for traversing (as usual, it takes a time to travel over an edge).
- Other edges (double blue) are tunnels:

A routing open shop problem with tunnels



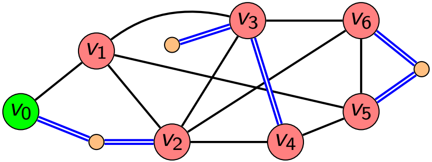
- v_0 is the depot.
- Some nodes (red ones plus depot) contain jobs (perhaps multiple jobs per node). Orange nodes do not contain jobs.
- Some edges (black) are used for traversing (as usual, it takes a time to travel over an edge).
- Other edges (double blue) are tunnels:
  - Can be used for traveling as black ones (travelling times are also involved).

A routing open shop problem with tunnels



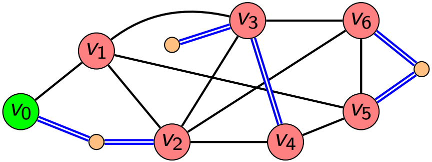
- v_0 is the depot.
- Some nodes (red ones plus depot) contain jobs (perhaps multiple jobs per node). Orange nodes do not contain jobs.
- Some edges (black) are used for traversing (as usual, it takes a time to travel over an edge).
- Other edges (double blue) are tunnels:
  - Can be used for traveling as black ones (travelling times are also involved).
  - Contain single job each (processing times!).

A routing open shop problem with tunnels



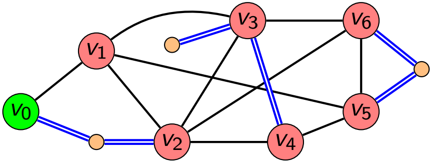
- v_0 is the depot.
- Some nodes (red ones plus depot) contain jobs (perhaps multiple jobs per node). Orange nodes do not contain jobs.
- Some edges (black) are used for traversing (as usual, it takes a time to travel over an edge).
- Other edges (double blue) are tunnels:
  - Can be used for traveling as black ones (travelling times are also involved).
  - Contain single job each (processing times!).
  - Any number of machines can travel over a tunnel at once, but only one can process an operation at a time.

A routing open shop problem with tunnels



- v_0 is the depot.
- Some nodes (red ones plus depot) contain jobs (perhaps multiple jobs per node). Orange nodes do not contain jobs.
- Some edges (black) are used for traversing (as usual, it takes a time to travel over an edge).
- Other edges (double blue) are tunnels:
  - Can be used for traveling as black ones (travelling times are also involved).
  - Contain single job each (processing times!).
  - Any number of machines can travel over a tunnel at once, but only one can process an operation at a time.
  - Machine processes the tunnel while traveling over it. Travel and processing times are combined.

A routing open shop problem with tunnels



- v_0 is the depot.
- Some nodes (red ones plus depot) contain jobs (perhaps multiple jobs per node). Orange nodes do not contain jobs.
- Some edges (black) are used for traversing (as usual, it takes a time to travel over an edge).
- Other edges (double blue) are tunnels:
  - Can be used for traveling as black ones (travelling times are also involved).
  - Contain single job each (processing times!).
  - Any number of machines can travel over a tunnel at once, but only one can process an operation at a time.
  - Machine processes the tunnel while traveling over it. Travel and processing times are combined.
- The goal is to process all jobs and to return to the depot ASAP.

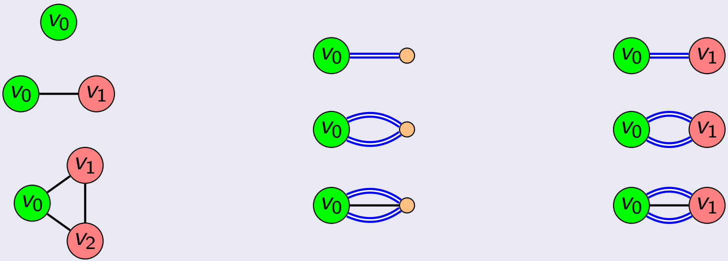
Ultimate instance reduction (for $m = 2$)

Theorem 1

For any instance I of $RO2||R_{\max}$, one can use job aggregation, chord removal and cycle/chain contractions to transform I into I' with at most two tunnels, such that $\bar{R}(I') = \bar{R}(I)$ and I' has one of the following structures.

Possible structures

The depot and v_2 contain at most one job each. The node v_1 contains at most 3.



The problem with tunnels in more detail

The standard lower bound?

Standard lower bound for $ROm||R_{\max}$

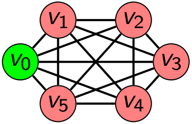
$$\bar{R} = \max\left\{ \ell_{\max} + T^*, \max_{v \in V}\left(d_{\max}(v) + 2\mathrm{dist}(v_0, v) \right) \right\}$$

The problem with tunnels in more detail

The standard lower bound?

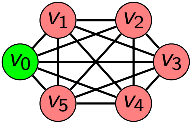
Standard lower bound for $ROm||R_{\max}$

$$\bar{R} = \max\left\{\ell_{\max} + T^*, \max_{v\in V}\left(d_{\max}(v) + 2\mathrm{dist}(v_0, v)\right)\right\}$$



Standard lower bound for $ROm||R_{\max}$

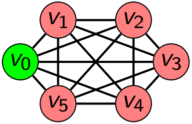
$$\bar{R} = \max\left\{\ell_{\max} + T^*, \max_{v \in V}\left(d_{\max}(v) + 2\mathrm{dist}(v_0, v)\right)\right\}$$



Traveling
salesman
problem

Standard lower bound for $ROm||R_{\max}$

$$\bar{R} = \max\left\{\ell_{\max} + T^*_{TSP}, \max_{v\in V}\left(d_{\max}(v) + 2\mathrm{dist}(v_0, v)\right)\right\}$$

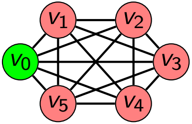


Traveling salesman problem

T^* is the TSP optimum.

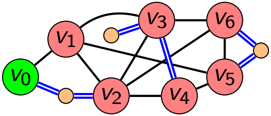
Standard lower bound for $ROm||R_{\max}$

$$\bar{R} = \max\left\{\ell_{\max} + T^*_{TSP}, \max_{v\in V}\left(d_{\max}(v) + 2\mathrm{dist}(v_0, v)\right)\right\}$$



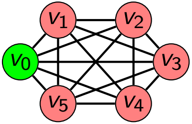
Traveling salesman problem

T^*_{TSP} is the TSP optimum.



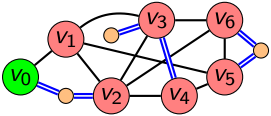
Standard lower bound for $ROm||R_{\max}$

$$\bar{R} = \max\left\{\ell_{\max} + T^*_{TSP}, \max_{v\in V}\left(d_{\max}(v) + 2\mathrm{dist}(v_0, v)\right)\right\}$$



Traveling salesman problem

T^*_{TSP} is the TSP optimum.

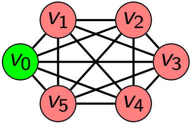


Rural postman problem

The problem with tunnels in more detail
The standard lower bound?

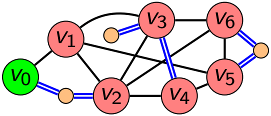
Standard lower bound for $ROm||R_{\max}$

$$\bar{R} = \max\left\{ \ell_{\max} + T^*_{TSP}, \max_{v \in V}\left(d_{\max}(v) + 2\mathrm{dist}(v_0, v) \right) \right\}$$



Traveling salesman problem

T^*_{TSP} is the TSP optimum.



Rural postman problem

T^*_{RPP} is the RPP optimum.

The standard lower bound for the problem with tunnels

Notation for $\overline{\overline{R}}Om||R_{\max}$

- $\mathcal{T} \subseteq E$ — the set of tunnels, T^*_{RPP} — the RPP optimum on G,
- $\forall \tau = [v, u] \in \mathcal{T}$:
  - $|\tau|$ — travel time over τ,
  - $j(\tau)$ — (extended) job, located on τ,
  - $\mathrm{dist}(v_0, \tau) = \min\{\mathrm{dist}(v_0, v), \mathrm{dist}(v_0, u)\}$.

The standard lower bound for the problem with tunnels

Notation for $\overline{\overline{R}}Om||R_{\max}$

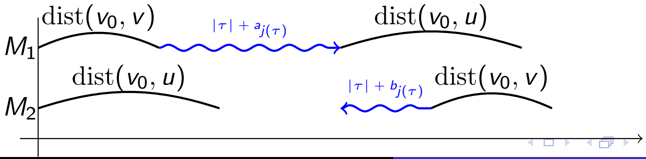
- $\mathcal{T} \subseteq E$ — the set of tunnels, T^*_{RPP} — the RPP optimum on G,
- $\forall \tau = [v, u] \in \mathcal{T}$:
  - $|\tau|$ — travel time over τ,
  - $j(\tau)$ — (extended) job, located on τ,
  - $\mathrm{dist}(v_0, \tau) = \min\{\mathrm{dist}(v_0, v), \mathrm{dist}(v_0, u)\}$.

$$\overline{\overline{R}} = \max\Big\{ \ell_{\max} + T^*_{RPP}, \max_{v \in V} \left(d_{\max}(v) + 2\mathrm{dist}(v_0, v)\right), \max_{\tau \in \mathcal{T}} \left(d_{j(\tau)} + 2|\tau| + 2\mathrm{dist}(v_0, \tau)\right) \Big\}.$$

Notation for $\overline{\overline{R}}Om||R_{\max}$

- $\mathcal{T} \subseteq E$ — the set of tunnels, T^*_{RPP} — the RPP optimum on G,
- $\forall \tau = [v, u] \in \mathcal{T}$:
  - $|\tau|$ — travel time over τ,
  - $j(\tau)$ — (extended) job, located on τ,
  - $\mathrm{dist}(v_0, \tau) = \min\{\mathrm{dist}(v_0, v), \mathrm{dist}(v_0, u)\}$.

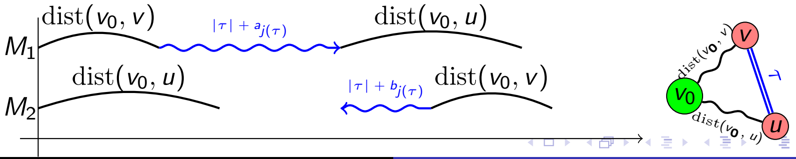
$$\overline{\overline{R}} = \max\Big\{\ell_{\max} + T^*_{RPP}, \max_{v\in V}\left(d_{\max}(v) + 2\mathrm{dist}(v_0, v)\right), \max_{\tau\in\mathcal{T}}\left(d_{j(\tau)} + 2|\tau| + 2\mathrm{dist}(v_0, \tau)\right)\Big\}.$$



Notation for $\overline{\overline{R}}Om||R_{\max}$

- $\mathcal{T} \subseteq E$ — the set of tunnels, T^*_{RPP} — the RPP optimum on G,
- $\forall \tau = [v, u] \in \mathcal{T}$:
  - $|\tau|$ — travel time over τ,
  - $j(\tau)$ — (extended) job, located on τ,
  - $\mathrm{dist}(v_0, \tau) = \min\{\mathrm{dist}(v_0, v), \mathrm{dist}(v_0, u)\}$.

$$\overline{\overline{R}} = \max\Big\{\ell_{\max} + T^*_{RPP}, \max_{v\in V}\left(d_{\max}(v) + 2\mathrm{dist}(v_0, v)\right), \max_{\tau\in\mathcal{T}}\left(d_{j(\tau)} + 2|\tau| + 2\mathrm{dist}(v_0,\tau)\right)\Big\}.$$

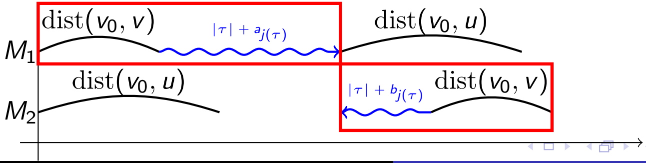


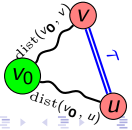
The standard lower bound for the problem with tunnels

Notation for $\overline{\overline{R}}Om||R_{\max}$

- $\mathcal{T} \subseteq E$ — the set of tunnels, T^*_{RPP} — the RPP optimum on G,
- $\forall \tau = [v, u] \in \mathcal{T}$:
    - $|\tau|$ — travel time over τ,
    - $j(\tau)$ — (extended) job, located on τ,
    - $\mathrm{dist}(v_0, \tau) = \min\{\mathrm{dist}(v_0, v), \mathrm{dist}(v_0, u)\}$.

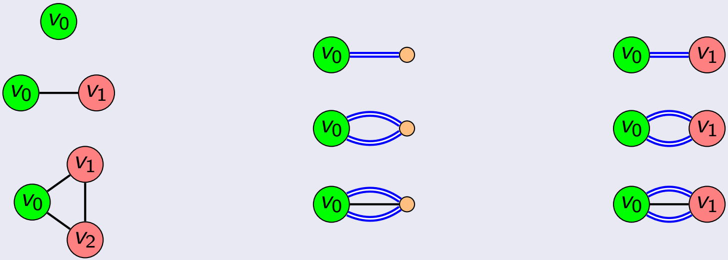
$$\overline{\overline{R}} = \max\Big\{ \ell_{\max} + T^*_{RPP}, \max_{v \in V} \left(d_{\max}(v) + 2\mathrm{dist}(v_0, v) \right), \max_{\tau \in \mathcal{T}} \left(d_{j(\tau)} + 2|\tau| + 2\mathrm{dist}(v_0, \tau) \right) \Big\}.$$





Overview of "simple" structures

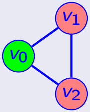
Possible structures from Theorem 1

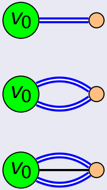


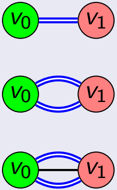
Overview of “simple” structures

Possible structures from Theorem 1

Polytime, $OPT = \bar{\bar{R}}$
[Gonzalez,Sahni 1976]





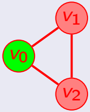


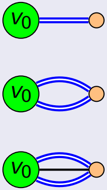
Overview of "simple" structures

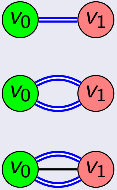
Possible structures from Theorem 1

Polytime, $OPT = \overline{\overline{R}}$
[Gonzalez,Sahni 1976]

NP-hard, $OPT \leqslant \frac{6}{5}\overline{\overline{R}}$
[Averbakh, Berman, Ch 2006]





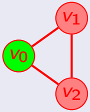


Overview of “simple” structures

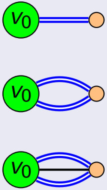
Possible structures from Theorem 1

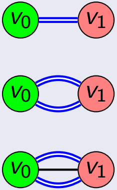
Polytime, $OPT = \overline{\overline{R}}$
[Gonzalez,Sahni 1976]

NP-hard, $OPT \leqslant \frac{6}{5}\overline{\overline{R}}$
[Averbakh, Berman, Ch 2006]



$OPT \leqslant \frac{6}{5}\overline{\overline{R}}$ [Ch, Lgotina 2016]



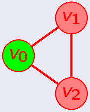


Overview of “simple” structures

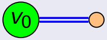
Possible structures from Theorem 1

Polytime, $OPT = \overline{\overline{R}}$ [Gonzalez,Sahni 1976]

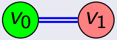
NP-hard, $OPT \leqslant \frac{6}{5}\overline{\overline{R}}$ [Averbakh, Berman, Ch 2006]

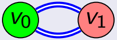


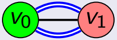
$OPT \leqslant \frac{6}{5}\overline{\overline{R}}$ [Ch, Lgotina 2016]



Polytime, $OPT = \overline{\overline{R}}$ [Ch, Shigina 2023]





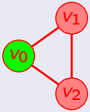


Overview of “simple” structures

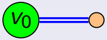
Possible structures from Theorem 1

Polytime, $OPT = \overline{\overline{R}}$ [Gonzalez,Sahni 1976]

NP-hard, $OPT \leqslant \frac{6}{5}\overline{\overline{R}}$ [Averbakh, Berman, Ch 2006]

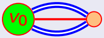


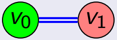
$OPT \leqslant \frac{6}{5}\overline{\overline{R}}$ [Ch, Lgotina 2016]

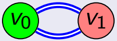


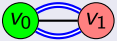
Polytime, $OPT = \overline{\overline{R}}$ [Ch, Shigina 2023]

NP-hard, $OPT \leqslant \frac{6}{5}\overline{\overline{R}}$ [Ch, Shigina 2023]









General two-machine problem with tunnels

Theorem 2

There exist a linear 2-approximation algorithm for $\overline{\overline{R}}O2||R_{\max}$ problem.

General two-machine problem with tunnels

Theorem 2

There exist a linear 2-approximation algorithm for $\overline{\overline{R}}O2||R_{\max}$ problem.

Algorithm

1. Find a $\frac{3}{2}$-approximate solution for the underlying RPP. Enumerate jobs according to that solution.
2. Let machine M_1 process jobs in order of that enumeration, while M_2 process jobs in opposite order.
3. If collision occurs, try to resolve it in two possible ways and choose the best.

Open questions

1. Is it true, that for $RO2||R_{\max}$ $OPT \leqslant \frac{5}{4}\bar{R}$? (We know, that $OPT \leqslant \frac{4}{3}\bar{R}$ [Ch, Kononov, Sevastyanov 2013])
2. Is it true, that for special cases $\overline{\overline{R}}O2||R_{\max}$ from Theorem 1 $OPT \leqslant \frac{5}{4}\bar{R}$? (If it is true, number 1 follows)
3. Find a better approximation for $\overline{\overline{R}}O2||R_{\max}$

Open questions

1. Is it true, that for $RO2||R_{\max}$ $OPT \leqslant \frac{5}{4}\bar{R}$? (We know, that $OPT \leqslant \frac{4}{3}\bar{R}$ [Ch, Kononov, Sevastyanov 2013])
2. Is it true, that for special cases $\overline{\overline{R}}O2||R_{\max}$ from Theorem 1 $OPT \leqslant \frac{5}{4}\bar{R}$? (If it is true, number 1 follows)
3. Find a better approximation for $\overline{\overline{R}}O2||R_{\max}$

Thank you for your attention!