Исследование задачи составления расписания выполнения заказов клиентов с двумя критериями

Захаров Алексей

Институт математики им. С.Л. Соболева СО РАН (Омский филиал)

Семинар “Модели и алгоритмы для задач составления расписаний”

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Постановка задачи

Input data

m is the number of customers
n is the number of products
$p_{ij} \geq 0$ is the duration of producing product j for customer i
$s_{jj'} \geq 0$ is the setup time from product j to product j'
s'_j is the initial setup to product j
d_i is the due date for customer i
q_i is the weight for customer i

Solution representation

We define operation as a pair of customer and product type (i, j), $i = 1, \dots, m$, $j = 1, \dots, n$. Operations should be scheduled without preemptions.

Предыдущие исследования

Hazir, O., Gunalay, Y., Erel, E. (2008). Customer order scheduling problem: a comparative metaheuristics study. The International Journal of Advanced Manufacturing Technology, 37, 589-598.

Erel, E., Ghosh, J. B. (2007). Customer order scheduling on a single machine with family setup times: Complexity and algorithms. Applied Mathematics and Computation, 185(1), 11-18.

Cetinkaya, F. C., Yeloglu, P., Catmakas, H. A. (2021). Customer order scheduling with job-based processing on a single-machine to minimize the total completion time.

Kovalenko, Y. V., Zakharov, A. O. (2020). The Pareto set reduction in bicriteria customer order scheduling on a single machine with setup times. In Journal of Physics: Conference

Предыдущие исследования

B.M.T. Lin, A.V. Kononov (2007) Customer order scheduling to minimize the number of late jobs, Euro J Oper Res

Bruno de Athayde Prata, Carlos Diego Rodrigues, Jose Manuel Framinan (2021) Customer order scheduling problem to minimize makespan with sequence-dependent setup times, Comput & Ind Engin

Z. Shi, L. Wang, P. Liu and L. Shi (2017) Minimizing Completion Time for Order Scheduling: Formulation and Heuristic Algorithm, IEEE Transactions on Automation Science and Engineering

Jose M. Framinan, Paz Perez-Gonzalez (2017) New approximate algorithms for the customer order scheduling problem with total completion time objective, Comp & Oper Res

Модель ЦЛП

$M = \{1, \dots, m\}$, $N = \{1, \dots, n\}$, $K = \{1, \dots, nm\}$.
Operation o is a pair (i, j).
We denote the set of all operations as O.

$$x_{ok} = \begin{cases} 1, & \text{if operation } o \in O \text{ is placed in position } k \; inK, \\ 0 & \text{otherwise,} \end{cases}$$

$t_k^f \geq 0$ is the completion time of an operation in position $k \in K$,

$$T_i \geq 0 \text{ is the completion time of the service of customer } i \in M,$$

$$k \in K, \; o \in O, \; i \in M.$$

Модель ЦЛП

Ограничения

$$\sum_{k\in K} x_{ok} = 1,\ o\in O, \tag{1}$$

$$\sum_{o\in O} x_{ok} = 1,\ k\in K, \tag{2}$$

$$t_1^f \geq \sum_{o\in O} x_{o1}(p_o + s'_o), \tag{3}$$

$$t_k^f \geq t_{k-1}^f + p_o + \sum_{o'\in O} x_{o',k-1} s_{o'o} - H(1 - x_{ok}), \tag{4}$$

$$k = 2,\ldots,nm,\ o\in O,$$

$$T_i \geq t_k^f - H(1 - x_{ok}),\ k\in K,\ o\in O, \tag{5}$$

$$T_i \geq 0,\ t_k^f \geq 0,\ x_{ok}\in\{0,1\},\ i\in M,\ k\in K, o\in O. \tag{6}$$

Критерии

Total completion time

$\sum_{i=1}^{m} T_i$

Weighted throughput

$\sum_{i \in O(\pi)} q_i$
$O(\pi)$ is the subset of customers, for which $T_i \leqslant d_i$ in π

Makespan

$\max_{i=1,\ldots,m} T_i$

Maximum lateness

$\max_{i=1,\ldots,m}\{T_i - d_i\}$

Двухкритериальная задача

Критерий

$f = (f_1, f_2)$
$f_1 = \sum_{i \in M} T_i, \quad f_2 = \max_{i \in M} T_i$

Множество допустимых решений

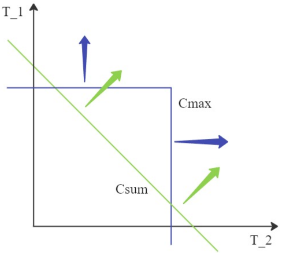
$X = \{x_{ok} \in \{0, 1\}, \ \forall o \in O, \forall k \in K : (1)–(6)\}.$
$Y = f(X)$

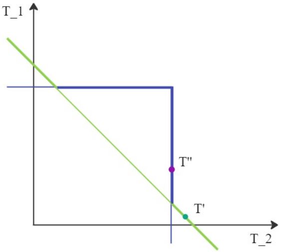
Множество Парето

$P(Y) = \{y^* \in Y | \nexists y \in Y : \ y \leq y^*\}$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \leq \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} 5 \\ 4 \end{pmatrix} \leq \begin{pmatrix} 5 \\ 7 \end{pmatrix}, \quad \begin{pmatrix} 3 \\ 1 \end{pmatrix} \ ? \ \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

Исследование множества Парето

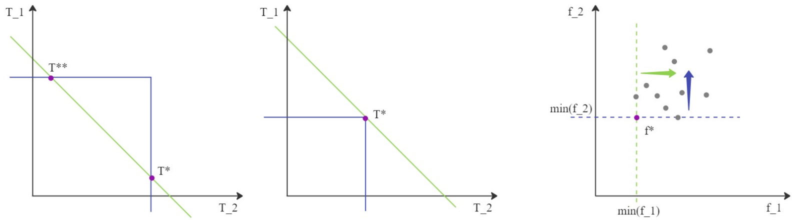




$$f_1 = \sum C_j, \quad f_2 = C_{\max}$$

$$\mathbf{T}' = \{T(x),\ x \in \arg\min_{x \in X} f_1(T(x))\},$$
$$\mathbf{T}'' = \{T(x),\ x \in \arg\min_{x \in X} f_2(T(x))\}.$$

Если $\mathbf{T}' \cap \mathbf{T}'' \neq \emptyset$, то $|P(Y)| = 1$.

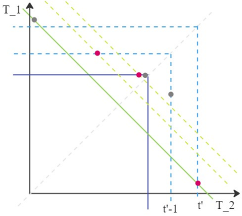


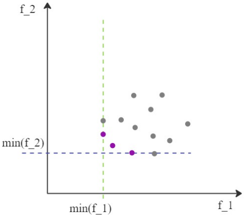
Общий случай

$$f_1 = \sum C_j, \quad f_2 = C_{\max}$$

$$\mathbf{T}' = \{T(x),\ x \in \arg\min_{x\in X} f_1(T(x))\},$$
$$\mathbf{T}'' = \{T(x),\ x \in \arg\min_{x\in X} f_2(T(x))\}.$$

Если $\mathbf{T}' \cap \mathbf{T}'' = \emptyset$, то $|P(Y)| > 1$.



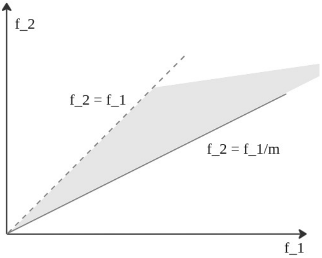


Утверждение

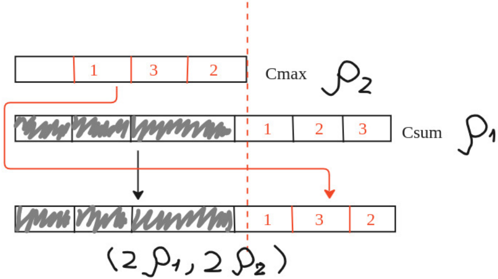
Множество Парето $P(Y)$ задачи (1)–(6) имеет следующую оценку сверху $|P(Y)| \le t' - \min_{x\in X} f_2 + 1$,
$t' = \min\{\max_{i\in M} T_i | \ \forall T \in \mathbf{T}'\}$.

Область допустимых значений

$$f_1 = \sum C_j, \quad f_2 = C_{\max}$$



Приближенное решение



$(2\rho_1, 2\rho_2)$ приближенное решение

Thank you for your attention!