Двухкритериальная модификация задачи составления расписаний с критерием суммарного взвешенного запаздывания и нечеткими исходными данными

А. Захаров, Ю. Захарова

Институт математики им. С.Л. Соболева

Исследование выполнено за счет гранта Российского научного фонда № 22-71-10015.

Fuzzy Set

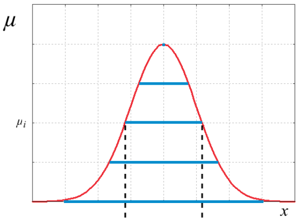
- U be a non-empty set of elements
- Fuzzy set A in U is the set of ordered pairs $\{u, \mu_A(u)\}$ for any $u \in U$
- $\mu_A(\cdot)$ is the membership function, $\mu_A(u) \in [0, 1]$
- $h(A) = \sup_{u \in U} \mu_A(u)$ is the height of a fuzzy set A
- If $h(A) = 1$, then A is called normal
- If $\mu_A(\lambda u + (1 - \lambda)w) \geqslant \min\{\mu_A(u), \mu_A(w)\}$ for $\forall u, w \in U$, $\lambda \in (0, 1)$, then A in linear space U is convex

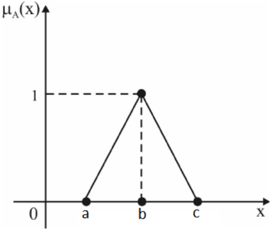
Fuzzy Number

A fuzzy number is convex normal fuzzy set in the set of real numbers $\mathbb{R}$.

Triangular fuzzy number

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a, \\ \frac{x-a}{b-a} & \text{if } a \leqslant x < b, \\ \frac{c-x}{c-b} & \text{if } b \leqslant x < c, \\ 0 & \text{if } x \geqslant c, \end{cases}$$





Previous Research

General Approach

- Zadeh L A 1965 Fuzzy sets
- Ekel P, Pedrycz W and Schinzinger R 1998 A general approach to solving a wide class of fuzzy optimization problems
- Chanas S and Kuchta D 1998 Discrete fuzzy optimization

Applications

- Slowinski R. and Hapke, M. 2000 Scheduling Under Fuzziness
- Huang Ch.-S., Huang Y.-Ch. and Lai P.-J. 2012 Modified genetic algorithms for solving fuzzy flow shop scheduling problems and their implementation with CUDA
- Zhang Y., Hu Q., Meng Z. and Ralescu, A. 2020 Fuzzy dynamic timetable scheduling for public transit
- Pang W., Wang K.-P., Zhou Ch.-G. and Dong, L.-J. 2004 Fuzzy Discrete Particle Swarm Optimization for Solving Traveling Salesman Problem
- Tang J., Pan Z., Fung R. Y. K. and Lau H. 2009 Vehicle routing problem with fuzzy time windows
- Nucci F. 2021 Multi-shift Single-Vehicle Routing Problem Under Fuzzy Uncertainty

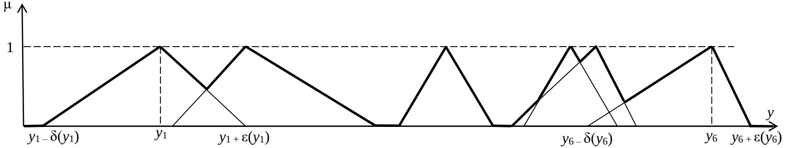
Problem Formulation

X is the set of feasible solutions
J is the set of parameters indices
$(p_j - \delta_j, p_j, p_j + \varepsilon_j)$, $\varepsilon_j, \delta_j > 0$, $j \in J$ are the fuzzy input parameters
f is objective function (to be maximized, summed value of some set of input parameters values)
$Y = f(X)$ is the set of feasible outcomes

Membership function μ_f of f (principle of Zadeh)

$$\mu_f(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \{\alpha \in [0,1] \mid \mu(x) = \alpha\}, & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases}$$

Problem Formulation: Example of μ_f



Bi-criteria Problem $< Y, g >$

Problem

Y is the the set of feasible alternatives
$Y = [\min_{x \in X} f_x - \delta_{f_x}, \max_{x \in X} f_x + \varepsilon_{f_x}]$
$g = (f, \mu_f)$ is the vector criterion,
$g(y) = (f(y), \mu_f(y))$ for $y \in Y$

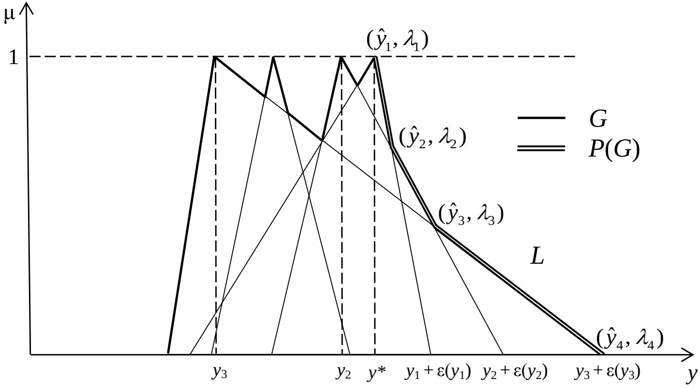
The Pareto set

$P_g(Y) = \{y \in Y \mid \nexists y^* \in Y : g(y^*) \ \geq \ g(y) \ \}$
$P(G) = g(P_g(Y))$

Construction of the Pareto Set

$G = g(Y)$

The Pareto set $P(G)$ is the most right polygonal line L, which is constructed by the right sides of TFNs corresponding to the maximum and near-maximum values of the function $f(x)$.



The Pareto Set Reduction

Noghin V.D.: Reduction of the Pareto Set: An Axiomatic Approach (2018)

Bi-criteria choice problem $< Y, g, \succ >$

$< Y, g = (g_1, g_2) > +$ asymmetric binary preference relation of the decision maker (DM) $\succ$:
$g(y^*) \succ g(y)$ means that the DM prefers the value y^* to y.

Axioms of "reasonable" choice

$\succ$ is irreflexive, transitive,
$\succ$ is invariant with respect to a linear positive transformation,
$\succ$ is compatible with criteria g_1, g_2,
exclusion axiom.

Elementary information quantum

We say that there is an *elementary information quantum* about the DM's preference relation $\succ$ with given coefficient of compromise $\theta \in (0,\ 1)$ if vector $z' \in \mathbb{R}^2$ such that $z'_i = 1 - \theta > 0$ and $z'_j = -\theta < 0$ satisfies the expression $z' \succ 0_2$, where $0_2 = (0, 0)$. Here $i, j \in \{1, 2\}$, $i \neq j$.

The Pareto Set Reduction

$$\begin{pmatrix} 0.8 \\ -0.2 \end{pmatrix} \ngeq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0.8 \\ -0.2 \end{pmatrix} \nleq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\text{Let } \begin{pmatrix} 0.8 \\ -0.2 \end{pmatrix} \succ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \theta = 0.2$$

The Edgeworth–Pareto Principle

If axioms of "reasonable" choice hold, then for any set of selectable outcomes $C(G) \subseteq P(G)$.

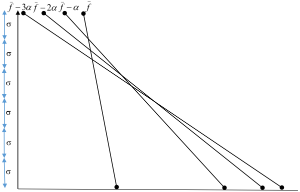
Theorem of the reduction

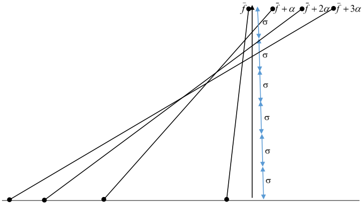
An elementary information quantum is given $f_1 \longrightarrow f_2 : \theta$. Then for any set of selectable outcomes $C(G)$ the inclusions
$C(G) \subseteq \hat{P}(G) \subseteq P(G)$, $\hat{P}(G) = g(P_{\hat{g}}(G))$,
$\hat{g}_1 = g_1, \hat{g}_2 = \theta g_1 + (1 - \theta) g_2$.

The Pareto set corresponds to a polygonal line L, which will be the southwestern border of some convex set. The polygonal line L is defined by N lines $g_2 = a^{(i)} - k^{(i)} g_1$, $i = 1, \dots, N$, so that $k^{(1)} \geq k^{(2)} \geq \dots \geq k^{(N)}$.

$g_1 \to g_2 : \theta$ or $g_2 \to g_1 : \theta$

For each value of $\theta \in (0, 1)$ we identify the cardinality of the Pareto set reduction. We specify the subintervals of interval $(0, 1)$, each of them gives some reduction of the Pareto set. Bounds of subintervals are formed by ratios between slope coefficients $k^{(i)}$, $i = 1, \dots, N$.





Theorem of the Reduction. Summed Objective and TFN

$$L = \{l^{(i)} \mid g_2 = a^{(i)} - k^{(i)} g_1, i = 1, \dots, p, k^{(1)} \geqslant k^{(2)} \geqslant \dots \geqslant k^{(p)}\}.$$

Theorem

Let $g_1 \to g_2 \colon \theta$, and the Pareto set is formed by polygonal line L.
1) *If $\theta < \frac{k^{(p)}}{k^{(p)}+1}$, then the reduction of the Pareto set $\widehat{P}(G)$ coincides with the Pareto set $P(G)$.*
2) *If $\frac{k^{(i)}}{k^{(i)}+1} \leqslant \theta < \frac{k^{(i-1)}}{k^{(i-1)}+1}$, then the reduction of the Pareto set $\widehat{P}(G)$ contains elements of segments $l^{(1)}, \dots, l^{(i-1)}$ and the rightmost element of L, $i = p, \dots, 2$.*
3) *If $\theta \geqslant \frac{k^{(1)}}{k^{(1)}+1}$, then the reduction of the Pareto set consists of one element.*

Here elements of segment we understand in terms of $[\dots)$, i. e. including the left point and excluding the right point subject to the first coordinate.

Scheduling Problem $1|r_j, d_j, w_j| \sum_j w_j T_j$

Input data

n is the number of jobs;
$p_j \geq 0$ is the duration of job j;
$r_j \geq 0$ is the release date of job j;
$d_j \geq 0$ is the due date of job j;
$w_j \geq 0$ is the weight of job j.

Criteria

C_j is the completion time of job j;
$T_j = \max\{0; C_j - d_j\}$ is the tardiness of job j;
$U_j = 1$ if $C_j > d_j$ and $U_j = 0$ otherwise;
the total weighted tardiness $\sum_j w_j T_j$;
the weighted number of tardy jobs $\sum_j w_j U_j$.

The problem is NP-hard.

MIP-model 1

$$x_{ij} = \begin{cases} 1 & \text{if job } j \text{ is placed in position } k, \\ 0 & \text{otherwise.} \end{cases}$$

$$f = \sum_{k=1}^{n} T_k \to \min, \quad (1)$$

$$\sum_{j=1}^{n} x_{jk} = 1, \ k = 1, \ldots, n, \quad (2)$$

$$\sum_{k=1}^{n} x_{jk} = 1, \ j = 1, \ldots, n, \quad (3)$$

$$T_k \geq \sum_{l=1}^{k} \sum_{j=1}^{n} x_{jk} p_j - \sum_{j=1}^{n} x_{jk} d_j, \ k = 1, \ldots, n, \quad (4)$$

$$x_{jk} \in \{0, 1\}, \ T_k \geq 0, \ j, k = 1, \ldots, n. \quad (5)$$

MIP-model 1

$$x_{ij} = \begin{cases} 1 & \text{if job } j \text{ is placed in position } k, \\ 0 & \text{otherwise.} \end{cases}$$

$$f = \sum_{j=1}^{n} w_j T_j \to \min, \tag{6}$$

$$\sum_{j=1}^{n} x_{jk} = 1, \;\; k = 1, \ldots, n, \tag{7}$$

$$\sum_{k=1}^{n} x_{jk} = 1, \;\; j = 1, \ldots, n, \tag{8}$$

$$T_k \geq \sum_{l=1}^{k} \sum_{j=1}^{n} x_{jk} p_j - \sum_{j=1}^{n} x_{jk} d_j, \;\; k = 1, \ldots, n, \tag{9}$$

$$T_j \geq T_k - T_{\max}(1 - x_{jk}), \;\; j, k = 1, \ldots, n, \tag{10}$$

$$x_{jk} \in \{0, 1\}, \;\; T_k \geq 0, \;\; T_j \geq 0, \;\; j, k = 1, \ldots, n. \tag{11}$$

MIP-model 2

$$x_{ij} = \begin{cases} 1 & \text{if job } i \text{ immediately precedes job } j, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if job } i \text{ precedes job } j, \\ 0 & \text{otherwise.} \end{cases}$$

$$f = \sum_{j=1}^{n} w_j T_j \to \min, \tag{12}$$

$$\sum_{j=1}^{n+1} x_{ij} = 1, \ i = 0, \dots, n, \tag{13}$$

$$\sum_{i=0}^{n} x_{ij} = 1, \ j = 1, \dots, n+1, \tag{14}$$

$$\sum_{i=1}^{n} y_{iq} - \sum_{i=1}^{n} y_{ik} + (n+2) \leq (n+1), \ q, k = 1, \dots, n, \tag{15}$$

$$T_j \geq \sum_{i=0}^{n} y_{ij} p_i + p_j - d_j, \ j = 1, \dots, n, \tag{16}$$

$$x_{ij} \in \{0,1\}, \ y_{ij} \in \{0,1\}, \ T_j \geq 0, \ i, j = 1, \dots, n. \tag{17}$$

Fuzzy Problem Formulation

Fuzzy Input Data

$(w_j - \delta_j, w_j, w_j + \varepsilon_j)$ is Triangular Fuzzy Number (TFN).

$$\mu_{w_j}(w) = \begin{cases} 0 & \text{if } w < w_j - \delta_j, \\ \frac{w - w_j + \delta_j}{\delta_j} & \text{if } w_j - \delta_j \leqslant w < w_j, \\ \frac{w_j + \varepsilon_j - w}{\varepsilon_j} & \text{if } w_j \leqslant w < w_j + \varepsilon_j, \\ 0 & \text{if } w \geqslant w_j + \varepsilon_j, \end{cases}$$

Membership Function

$$\mu_x(y) = \begin{cases} 0 & \text{if } y < f_x - \delta_{f_x}, \\ \frac{y - f_x + \delta_{f_x}}{\delta_{f_x}} & \text{if } f_x - \delta_{f_x} \leqslant y < f_x, \\ \frac{f_x + \varepsilon_{f_x} - y}{\varepsilon_{f_x}} & \text{if } f_x \leqslant y < f_x + \varepsilon_{f_x}, \\ 0 & \text{if } y \geqslant f_x + \varepsilon_{f_x}, \end{cases}$$

where $f_x = \sum_{j=1}^{n} w_j T_j$, $\delta_{f_x} = \sum_{j=1}^{n} \delta_j T_j$, $\varepsilon_{f_x} = \sum_{j=1}^{n} \varepsilon_j T_j$.

Crisp Bi-criteria Problem

Membership Function

$\mu_f(y) = \sup_{x \in X}\{\alpha \in [0,1] \mid \mu_x(y) = \alpha\}$

Two Criteria

Objective function and its membership function represent the criteria of crisp problem, i.e. we have vector criterion $g = (f, \mu_f)$, such that $g(y) = (y, \mu_f(y))$ for all $y \in [\min_{x \in X}(f_x - \delta_{f_x}), \max_{x \in X}(f_x + \varepsilon_{f_x})]$.

Approximation of the Pareto Set

1: Find optimal solution of model (6)-(11) for TWTP. Let f_1^* denote the optimal objective value, and triplet $(f_1^* - \delta_1^*, f_1^*, f_1^* + \varepsilon_1^*)$ is the corresponding TFN. Put the current iteration number $iter = 1$.

2: Repeat until $iter \le m_{\max}$.

   2.1 Put $iter = iter + 1$.

   2.2 Find optimal solution of model (6)-(11) for TWTP with additional constraint $f \ge f_{iter-1}^* + 1$.

   2.3 Let f_{iter}^* denote the obtained optimal objective value, and triplet $(f_{iter}^* - \delta_{iter}^*, f_{iter}^*, f_{iter}^* + \varepsilon_{iter}^*)$ is the corresponding TFN.

3: Find an approximation of the Pareto set from $\{(f_1^* - \delta_1^*, f_1^*, f_1^* + \varepsilon_1^*), \ldots, (f_{m_{\max}}^* - \delta_{m_{\max}}^*, f_{m_{\max}}^*, f_{m_{\max}}^* + \varepsilon_{m_{\max}}^*)\}$.

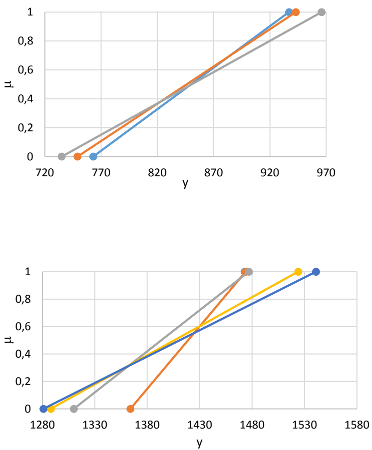
Computational Experiment

TWTP instances from OR-Library ($n = 40,\ n = 50$).

Values $\delta_j = \varepsilon_j$ are generated randomly with uniform distribution from interval $[\frac{w_{aver}}{8}, \frac{w_{aver}}{4})$, where w_{aver} is the average weight.

Series	n	|Approximation|		
		min	aver	max
Weighted tardiness	40	3	5	10
Weighted tardiness	50	4	6	12

TFNs giving the Pareto set approximation



Specific Series of Instances

The Pareto set consists of N TFNs:

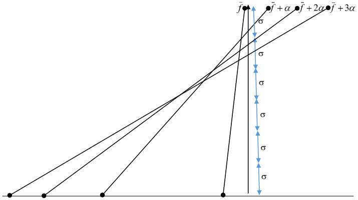
$(\bar f - \Delta \bar f, \bar f, \bar f + \Delta \bar f)$,

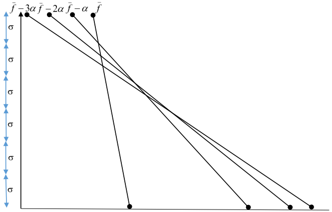
$\left((\bar f + i\theta) - \left(\Delta \bar f + \frac{\theta_{i-1}}{i\sigma} + \frac{\theta}{\sigma}\right), \bar f + i\theta, (\bar f + i\theta) + \left(\Delta \bar f + \frac{\theta_{i-1}}{i\sigma} + \frac{\theta}{\sigma}\right)\right)$,

$i = 1, \ldots, N-1$,

where $N = \lceil \frac{1}{\sigma} \rceil$, $0 < \sigma < 1$,

$\theta_0 = 0$, $\theta_i = \theta_{i-1}\left(1 + \frac{1}{i}\right) + \theta$, $i = 1, \ldots, N-1$.



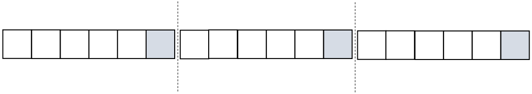


TWTP Problem

We select σ such that $N = \lceil \frac{1}{\sigma} \rceil = n$.

p_j are arbitrary crisp processing times
$d_j = d = \sum_{i=1}^{n} p_i - \min_{i=1,\dots,n} p_i$ is the common crisp due date

$\tilde{w}_j$ are fuzzy numbers represented weights of the form
$(w_j - \delta_j, w_j, w_j + \delta_j)$, where
$w_j = \bar{f} + j\theta$ for $j = 0, \dots, N-1$.
$\delta_j = \Delta\bar{f} + \frac{\theta}{\sigma} + \frac{\theta_{j-1}}{j\sigma}$, for $j = 1, \dots, N-1$, $\delta_0 = \Delta\bar{f}$.



Conclusions

- Fuzzy version of the single-machine scheduling problem is formulated as bi-objective one, where the total cost (main objective) and its membership function are optimized.
- An approximation of the Pareto set is constructed by the greedy algorithm using MIP-solver.
- Experimental evaluation on instances from TSPLIB library shows that the approximate set of the non-dominated solutions has a very low cardinality, which is appealing from the practical point of view.
- A series of instances with special structure of the Pareto set is constructed.

Thank you for your attention!