On analysis of models for customer order scheduling problem

Yu. Zakharova, A. Zakharov

Sobolev Institute of Mathematics SB RAS

The research was supported by Russian Science Foundation grant N 22-71-10015.

Problem Statement

Input data

m is the number of customers
n is the number of products
$p_{ij} \geq 0$ is the duration of producing product j for customer i
$s_{jj'} \geq 0$ is the setup time from product j to product j'
s'_j is the initial setup to product j
d_i is the due date for customer i
q_i is the weight for customer i

Solution representation

We define operation as a pair of customer and product type (i, j), $i = 1, \ldots, m$, $j = 1, \ldots, n$. Operations should be scheduled without preemptions. A set of feasible solution Π consists of permutations of operations $\pi = \{(i, j), i = 1, \ldots, m, j = 1, \ldots, n\}$.

Criteria

Makespan

$\max_{i=1,\dots,m} C_i$

Weighted throughput

$\sum_{i\in O(\pi)} q_i$
$O(\pi)$ is the subset of customers, for which $C_i \leqslant d_i$ in π

Total completion time

$\sum_{i=1}^{m} C_i$

Maximum lateness

$\max_{i=1,\dots,m}\{C_i - d_i\}$

Previous Research

Hazir, O., Gunalay, Y., Erel, E. (2008). Customer order scheduling problem: a comparative metaheuristics study. The International Journal of Advanced Manufacturing Technology, 37, 589-598.

Erel, E., Ghosh, J. B. (2007). Customer order scheduling on a single machine with family setup times: Complexity and algorithms. Applied Mathematics and Computation, 185(1), 11-18.

Cetinkaya, F. C., Yeloglu, P., Catmakas, H. A. (2021). Customer order scheduling with job-based processing on a single-machine to minimize the total completion time.

Kovalenko, Y. V., Zakharov, A. O. (2020). The Pareto set reduction in bicriteria customer order scheduling on a single machine with setup times. In Journal of Physics: Conference Series (Vol. 1546, No. 1, p. 012087). IOP Publishing.

Mathematical Programming Model 1

$$x_{ijk} = \begin{cases} 1, & \text{if product } j \text{ of customer } i \text{ is placed in position } k, \\ 0 & \text{otherwise;} \end{cases}$$

$t_k^f \geq 0$ is the completion time of an operation in position $k \in N$;

$T_i^f \geq 0$ is the completion time of the service of customer $i \in M$;

$$z_i = \begin{cases} 1, & \text{if the order of customer } i \text{ is made on time } (T_i^f \leq d_i), \\ 0 & \text{otherwise.} \end{cases}$$

Mathematical Programming Model 1 (constraints)

$$\sum_{k=1}^{|N|} x_{ijk} = 1,\ i = 1,\dots,m,\ j = 1,\dots,n, \tag{1}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = 1,\ k = 1,\dots,|N|, \tag{2}$$

$$t_1^f \geq \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij1}(p_{ij} + s'_j), \tag{3}$$

$$t_k^f \geq t_{k-1}^f + p_{ij} + \sum_{i'=1}^{m}\sum_{j'=1}^{n} x_{i'j',k-1} s_{j'j} - H(1 - x_{ijk}), \tag{4}$$

$$k = 2,\dots,|N|,\ i = 1,\dots,m,\ j = 1,\dots,n,$$

$$T_i^f \geq t_k^f - H(1 - x_{ijk}), \tag{5}$$

$$k = 1,\dots,|N|,\ i = 1,\dots,m,\ j = 1,\dots,n,$$

$$H(1 - z_i) \geq T_i^f - d_i,\ i = 1,\dots,m, \tag{6}$$

$$T_i^f \geq 0,\ t_k^f \geq 0,\ x_{ijk} \in \{0,1\},\ y_i \in \{0,1\}, \tag{7}$$

$$i = 1,\dots,m,\ j = 1,\dots,n,\ k = 1,\dots,|N|.$$

Mathematical Programming Model 2

$$x_{ijk} = \begin{cases} 1, & \text{if product } j \text{ of customer } i \text{ is placed in position } k, \\ 0 & \text{otherwise;} \end{cases}$$

$$b_{j'jk} = \begin{cases} 1, & \text{if arise a setup between products } j' \\ & \text{ and } j \text{ in position } k = 2, \dots, |N|, \\ 0 & \text{otherwise;} \end{cases}$$

$$z_{ik} = \begin{cases} 1, & \text{if customer } i \text{ is in the system in position } k, \\ 0 & \text{otherwise;} \end{cases}$$

$$U_{ijki'} = \begin{cases} 1, & \text{if both } x_{ijk} = 1 \text{ and } z_{i'k} = 1, \\ 0 & \text{otherwise;} \end{cases}$$

$$V_{j'jki'} = \begin{cases} 1, & \text{if both } b_{j'jk} = 1 \text{ and } z_{i'k} = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Mathematical Programming Model 2 (constraints)

$$\sum_{k=1}^{|N|} x_{ijk} = 1, \ i = 1, \ldots, m, \ j = 1, \ldots, n, \quad (8)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = 1, \ k = 1, \ldots, |N|, \quad (9)$$

$$k x_{ijk} \leq a_i, \quad i = 1, \ldots, m, \ j = 1, \ldots, n, \ k = 1, \ldots, |N|, \quad (10)$$

$$\sum_{k=1}^{|N|} z_{ik} = a_i, \ i = 1, \ldots, m, \quad (11)$$

$$z_{i,k+1} \leq z_{ik}, \ i = 1, \ldots, m, \ k = 1, \ldots, |N| - 1, \quad (12)$$

$$\sum_{i=1}^{m} x_{ijk} + \sum_{i=1}^{m} x_{i,j',k-1} - 1 \leq b_{j'jk}, \quad j, j' = 1, \ldots, n, \ k = 1, \ldots, |N|, \quad (13)$$

Operation is a pair (i, j).
We denote the set of all operations as O, and let O_i corresponds to operations of customer $i \in M$. We also add two auxiliary operations 0 and $nm + 1$ of zero duration.

$$x_{kl} = \begin{cases} 1, & \text{if operation } k \text{ immediately proceeds operation } l, \\ 0 & \text{otherwise;} \end{cases}$$

$\tilde{t}^f_k \geq 0$ is the completion time of operation $k \in O$;

$T^f_i \geq 0$ is the completion time of servicing customer $i \in M$.

Mathematical Programming Model 3 (constraints)

$$\sum_{l=1}^{nm+1} x_{kl} = 1, \ k = 0, 1, \ldots, nm, \tag{14}$$

$$\sum_{k=0}^{nm} x_{kl} = 1, \ l = 1, \ldots, nm+1, \tag{15}$$

$$\tilde{t}_k^f \geq \tilde{t}_l^f + s_{lk} + p_k - H(1 - x_{lk}), \ k, l = 0, \ldots, nm+1, \tag{16}$$

$$T_i^f \geq \tilde{t}_k^f, \ k \in O_i, \ i = 1, \ldots, m, \tag{17}$$

$$H(1 - z_i) \geq T_i^f - d_i, \ i = 1, \ldots, m, \tag{18}$$

$$T_i^f \geq 0, \ \tilde{t}_k^f \geq 0, \ x_{kl} \in \{0, 1\}, \ y_{kl} \in \{0, 1\}, z_i \in \{0, 1\}, \tag{19}$$

$$i = 1, \ldots, m, \ k, l = 1, \ldots, nm+1.$$

Mathematical Programming Model 4

$$x_{kl} = \begin{cases} 1, & \text{if operation } k \text{ immediately proceeds operation } l, \\ 0 & \text{otherwise;} \end{cases}$$

$$y_{kl} = \begin{cases} 1, & \text{if operation } k \text{ proceeds operation } l, \\ 0 & \text{otherwise.} \end{cases}$$

$$xy_{klq} = \begin{cases} 1, & \text{if } x_{kl} = 1 \text{ and } y_{lq} = 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$\sum_{l=1}^{nm+1} x_{kl} = 1,\ k = 0, 1, \ldots, nm, \tag{20}$$

$$\sum_{k=0}^{nm} x_{kl} = 1,\ l = 1, \ldots, nm+1, \tag{21}$$

$$\tilde{t}_q^f = \sum_{k,l=0}^{nm+1} xy_{klq} s_{kl} + \sum_{l=0}^{nm+1} y_{lq} p_l + p_q,\ q = 0, \ldots, nm+1, \tag{22}$$

$$T_i^f \geq \tilde{t}_k^f,\ k \in O_i,\ i = 1, \ldots, m, \tag{23}$$

$$xy_{klq} \geq y_{lq} + x_{kl} - 1,\ k, l, q = 0, \ldots, nm+1, \tag{24}$$

$$\sum_{k=1}^{nm} y_{kq} - \sum_{k=1}^{nm} y_{kl} + (nm+3) x_{ql} \leq nm+2,\ q, l = 1, \ldots, nm, \tag{25}$$

$$y_{kq} + y_{qk} = 1,\ q, k = 0, \ldots, nm+1,\ q \neq k. \tag{26}$$

$$H(1 - z_i) \geq T_i^f - d_i,\ i = 1, \ldots, m, \tag{27}$$

$$T_i^f \geq 0,\ \tilde{t}_k^f \geq 0,\ x_{kl} \in \{0,1\},\ y_{kl} \in \{0,1\}, z_i \in \{0,1\}, \tag{28}$$

$$i = 1, \ldots, m,\ k, l = 1, \ldots, nm+1$$

Additional constraints

$$x_{kl} \le y_{kl}, \ \ k,l = 0,\dots,nm+1, \qquad (29)$$

$$x_{kq} + x_{qk} \le 1, \ \ k,q = 0,\dots,nm+1, \qquad (30)$$

$$\sum_{k=0}^{nm} y_{kq} + \sum_{k=1}^{nm+1} y_{qk} = nm+1, \ \ q = 1,\dots,nm. \qquad (31)$$

Special case (zero setup)

$$t_k^f = \sum_{l=0}^{nm} y_{lk} p_l + p_k, \;\; k = 1, \ldots, nm + 1, \tag{32}$$

$$y_{kq} + y_{ql} + y_{lk} \leq 2, q, l, k = 1, \ldots, nm. \tag{33}$$

Thank you for your attention!