# О решении робастной задачи балансировки линии с параллельными операциями и интервальными длительностями обработки

**(материалы для доклада на XIV International Conference Optimization and Applications (OPTIMA-2023))**

**Борисовский П.А.**
*Институт математики им. С.Л. Соболева СО РАН*

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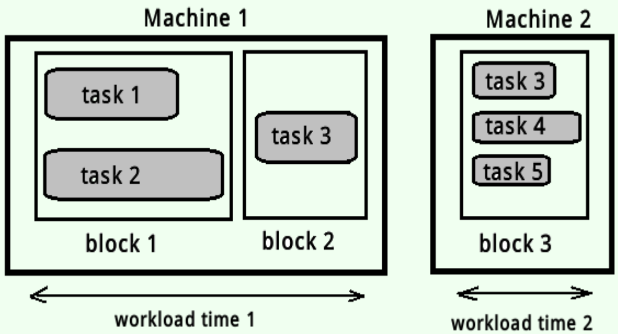
# Problem formulation

## Transfer line structure

- A set of *tasks* is given. Each task must be performed once.
- Tasks are grouped into *blocks* for a parallel execution. The block time is the maximal execution time of its tasks.
- Each *machine* can execute a sequence of blocks. A *workload time* of a machine is the sum of its blocks' times and it is limited by a given *cycle time* $\boldsymbol{T}$.
- A partial order on the tasks is given that defines *precedence* relations.



# Literature Review

Deterministic problem

**1.** Dolgui, A., Guschinsky, N., Levin, G.: On problem of optimal design of transfer lines with parallel and sequential operations. In: 7th IEEE International Conference on ETFA-1999, vol. 1, pp. 329–334.

Problem formulation, motivation, graph solution approach.

**2.** Guschinskaya, O., Gurevsky E., Dolgui, A., Eremeev, A.: Metaheuristic approaches for the design of machining lines. Int. J. Adv. Manuf. Technol **55**(1-4), 11–22 (2011)

GRASP, Genetic algorithm, test instances.

# Robust problem formulation

$\boldsymbol{V} = \{\mathbf{1}, \mathbf{2}, ..., \boldsymbol{n}\}$ is the set of all tasks;

$\boldsymbol{t} = (\boldsymbol{t}_1, ..., \boldsymbol{t}_n)$ is the vector of execution times;

$\widetilde{\boldsymbol{V}} \subseteq \boldsymbol{V}$ is the set of *uncertain* tasks, for which the execution times may deviate from their nominal values;

# Robust problem formulation

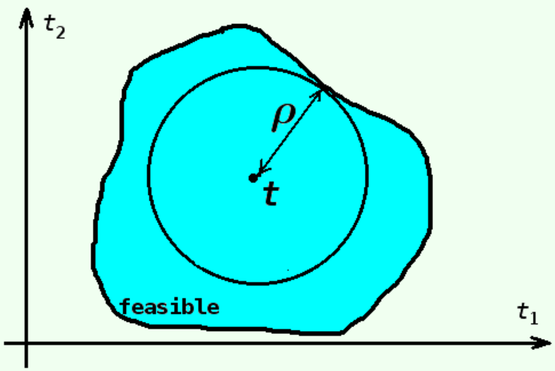
For some solution $\boldsymbol{S}$, a *stability radius* is the deviation supported by the solution which keeps its feasibility.

$$\boldsymbol{\rho}(\boldsymbol{S}, \boldsymbol{t}) = \max\{\boldsymbol{\varepsilon} \mid \forall \boldsymbol{\xi} \in \boldsymbol{B}(\boldsymbol{\varepsilon}) \text{ solution } \boldsymbol{S} \text{ stays feasible if}$$

$$\boldsymbol{t} \text{ is replaced by } \boldsymbol{t} + \boldsymbol{\xi}\}.$$

where

$$\boldsymbol{B}(\boldsymbol{\varepsilon}) = \{\boldsymbol{\xi} \in \boldsymbol{R}^n \mid \boldsymbol{\xi}_j > \mathbf{0} \text{ for uncertain tasks } \boldsymbol{j}, \text{ and } \|\boldsymbol{\xi}\| \leq \boldsymbol{\varepsilon}\}$$




In this work, $\boldsymbol{l_1}$-norm is used: $\|\boldsymbol{\xi}\|_1 = \sum_j \boldsymbol{\xi}_j$.

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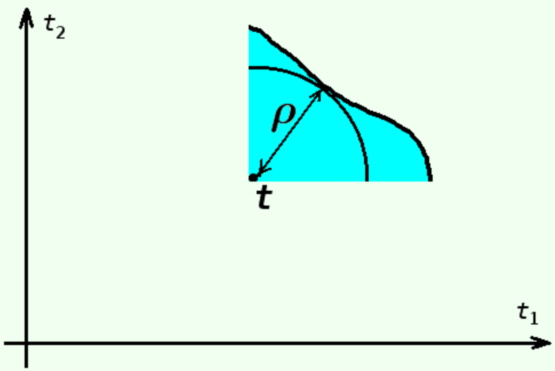


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# Literature Review

**1.** Pirogov A., Gurevsky E., Rossi A., Dolgui A.: Robust balancing of transfer lines with blocks of uncertain parallel tasks under fixed cycle time and space restrictions. Eur. J. Oper. Res **290**(3), 946–955 (2021)

Robust problem formulation, calculation of stability radius, MIP, test instances, greedy algorithm.

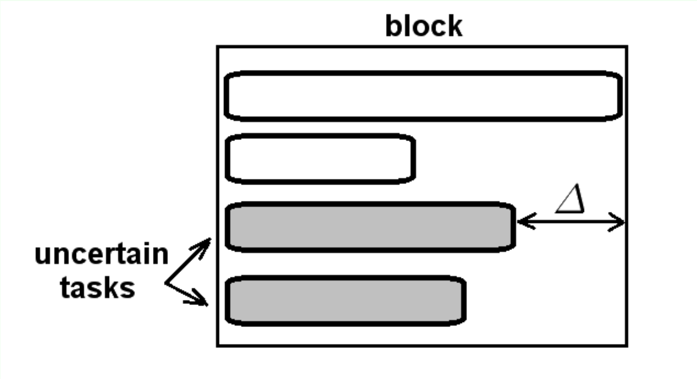
**2.** Borisovsky P., Battaïa O.: MIP-Based Heuristics for a Robust Transfer Lines Balancing Problem. In: OPTIMA 2021, LNCS, vol. 13078, pp. 123–135.

Inclusion/exclusion constraints, MIP based matheuristic.

# Calculation of stability radius

For any uncertain block $\boldsymbol{k}$ of machine $\boldsymbol{p}$ define a *save time* as the difference between the block working time $\boldsymbol{\tau_k}$ and the nominal processing time of its longest uncertain task, *i.e.*,

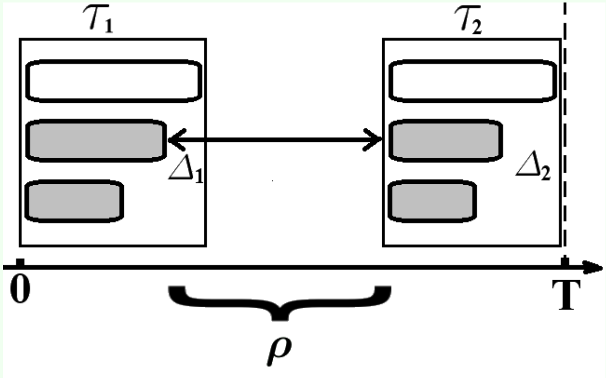
$$\boldsymbol{\Delta := \tau - \max_{j \in \widetilde{V}} t_j.}$$



## Calculation of stability radius, [1]

$$\rho = T - \sum \tau_k + \Delta_{\min}$$

i.e, take the block with the minimal $\Delta$ and extend its longest time until the total workload reaches $T$.




For a line with many machines $\rho = \min \rho_i$.

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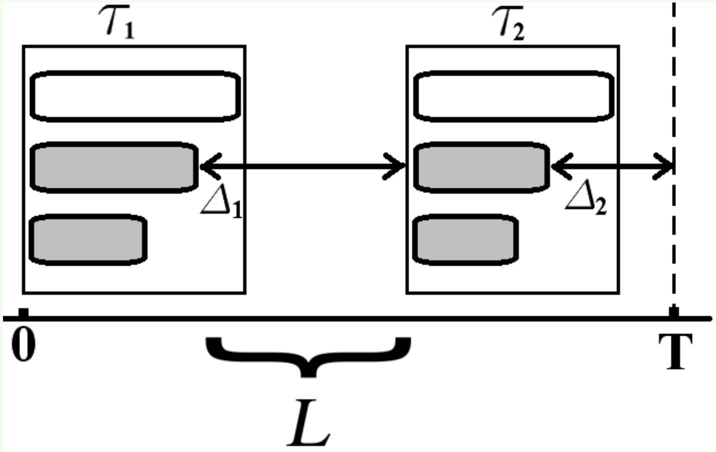
1. Pirogov A., Gurevsky E., Rossi A., Dolgui A.: Robust balancing of transfer lines with blocks of uncertain parallel tasks under fixed cycle time and space restrictions. EJOR **290**(3), 946–955 (2021)

# New formulation: interval processing times

In the previous formulation, no upper limit on the possible extra times was assumed.

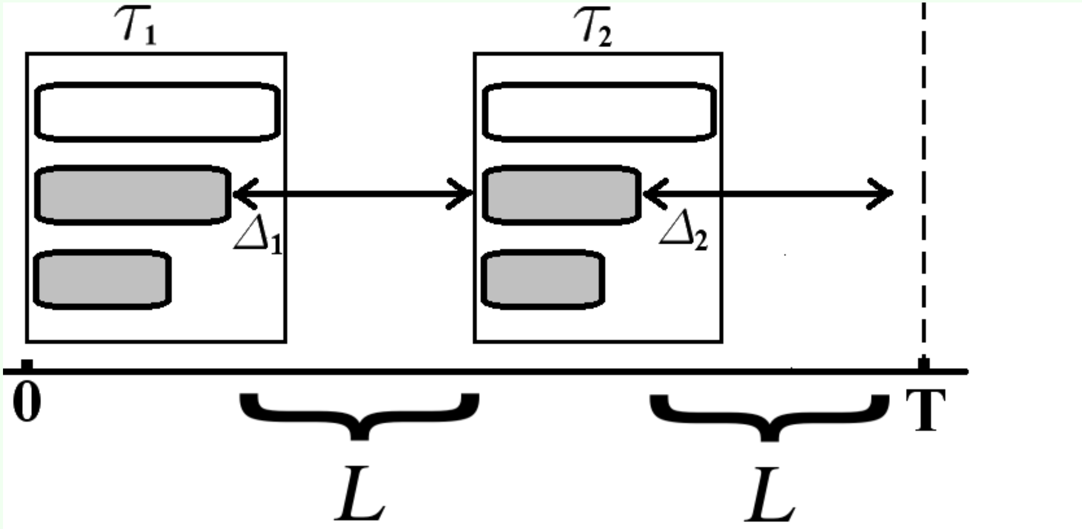
This may lead to improbable situations, where only one task has a large excess: $\boldsymbol{\xi} = (\mathbf{0}, .., \boldsymbol{\rho}, ..\mathbf{0})$

In this study, assume that $\boldsymbol{\xi_j} \leq \boldsymbol{L}$, i.e. the actual time is in $[\boldsymbol{t_j}, \boldsymbol{t_j} + \boldsymbol{L}]$.



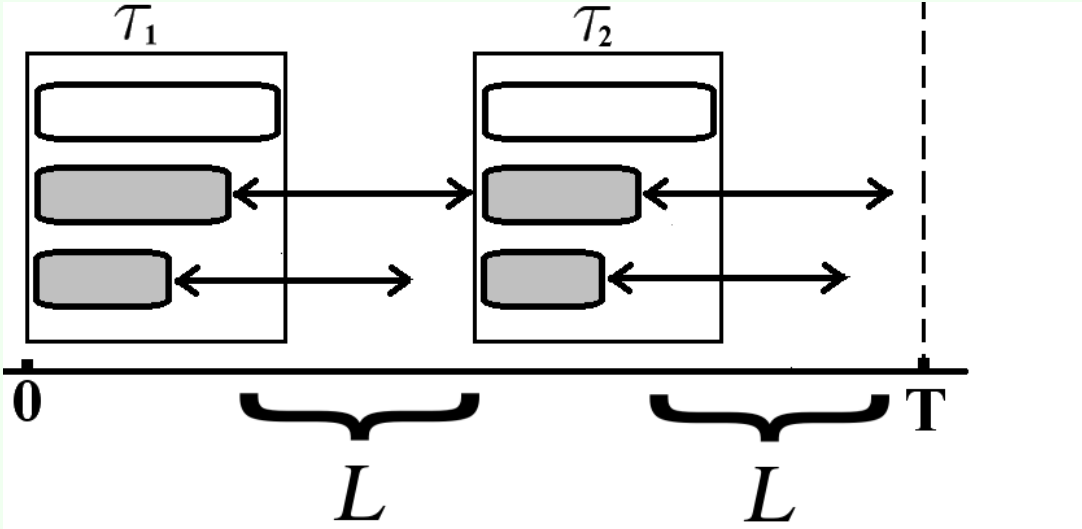
# Computing the stability radius

A special case: if $\boldsymbol{T}$ is large enough.



# New formulation

A special case: if $\boldsymbol{T}$ is large enough then the solution is always feasible and $\boldsymbol{\rho} = \boldsymbol{L}\tilde{\boldsymbol{n}}$.



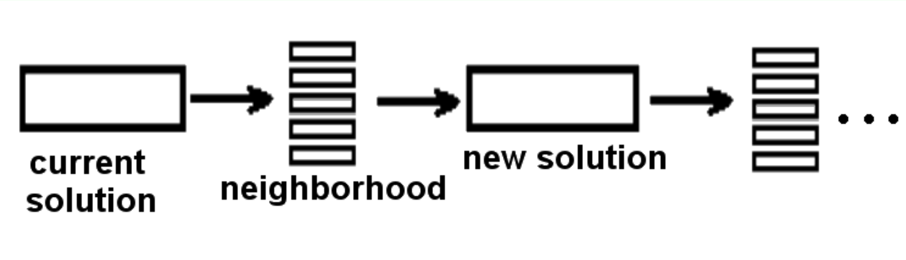
# Computing the stability radius

Computing the stability radius for one workstation

1. Sort blocks by increase of their save times
2. Let $\boldsymbol{\rho} := \mathbf{0}$, $\boldsymbol{R} := \boldsymbol{T} - \sum_{k=1}^{K} \boldsymbol{\tau}_k$.
3. For $\boldsymbol{k} = 1$ to $\boldsymbol{K}$ do:
   2.1. Let $\boldsymbol{a}_k := \boldsymbol{L} - \boldsymbol{\Delta}_k$.
   2.2. If $\boldsymbol{a}_k < \mathbf{0}$ then
   stop and **return** $\boldsymbol{L\tilde{n}}$.
   2.3. If $\boldsymbol{R} < \boldsymbol{a}_k$ then
   set $\boldsymbol{a}_k := \boldsymbol{R},\ \ \boldsymbol{\rho} := \boldsymbol{\rho} + \boldsymbol{\Delta}_k + \boldsymbol{a}_k$, stop and **return** $\boldsymbol{\rho}$.
   Else
   $\boldsymbol{R} := \boldsymbol{R} - \boldsymbol{a}_k$, $\boldsymbol{\rho} := \boldsymbol{\rho} + \boldsymbol{\Delta}_k + \boldsymbol{a}_k$.
4. **Return** $\boldsymbol{L\tilde{n}}$.

*Theorem.* The algorithm correctly finds the stability raduis.

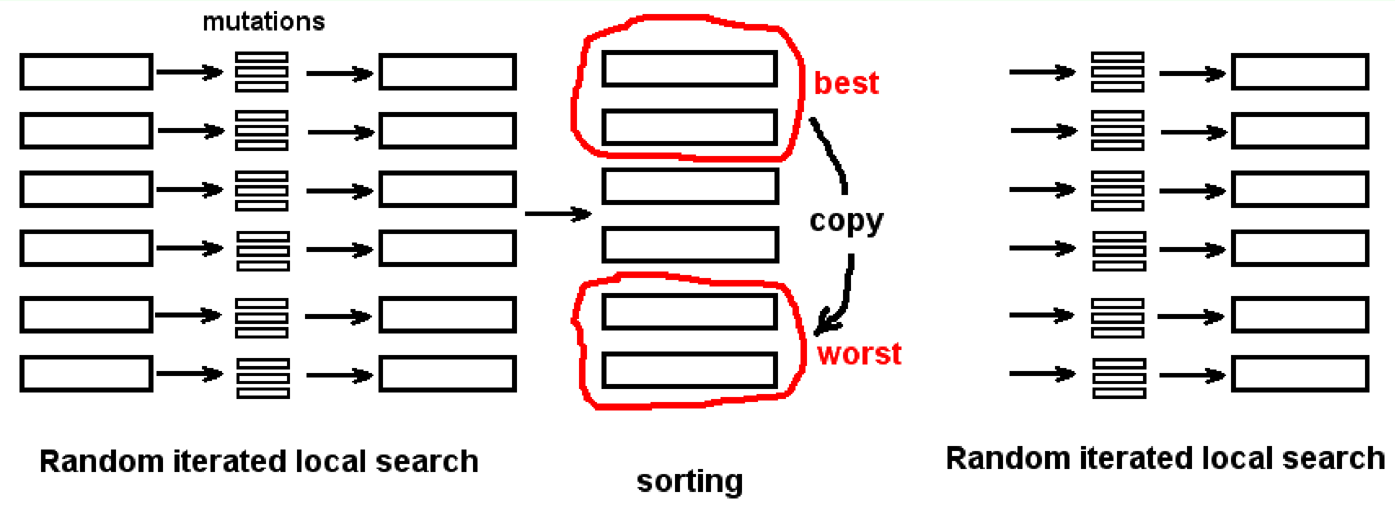
# Random Iterated Local Search




On each iteration:

1. Many times in parallel apply small random changes (*mutation*)
2. Select the best output and make it the new currect solution
3. On certain iterations, a *shaking* procedure is applied, which consists in several mutations in a sequence.

# Hybrid "Go with the Winners" algorithm




Mutations and evaluation of obtained solutions are done in parallel on a Graphics Processing Unit (GPU)


Aldous, D., Vazirani, U.: "Go with the winners" algorithms. In: Proc. 35th Annual Symposium on Foundations of Computer Science, Santa Fe, NM, USA, pp 492–501. IEEE (1994).

Borisovsky, P.: A parallel "Go with the winners" algorithm for some scheduling problems. Journal of Applied and Industrial Mathematics (in press)

# Computational Experiments

GPU Tesla V100 (1530 MHz, 5120 CUDA cores)
Test instances from (Pirogov et al., 2021).

| **Series S1** $n = 25, m = 5$ $L = 20$ | **optimize unlimited** | | **optimize limited** | |
|---|---|---|---|---|
| | $\rho_U^{best}$ | $\rho_L^{avg}$ | $\rho_U^{avg}$ | $\rho_L^{best}$ |
| S1.0 | 30.4 | 30.4 | 20.59 | 260 |
| S1.1 | 27.3 | 27.3 | 16.52 | 38.6 |
| S1.2 | 29.0 | 29.0 | 21.9 | 30.5 |
| S1.3 | 34.5 | 34.5 | 20.99 | 260 |
| S1.4 | 28.5 | 28.5 | 14.26 | 36.7 |
| S1.5 | 32.3 | 32.3 | 21.54 | 260 |
| S1.6 | 28.5 | 28.5 | 20.17 | 35.4 |
| S1.7 | 38.5 | 38.5 | 20.62 | 260 |
| S1.8 | 32.4 | 32.4 | 21.7 | 260 |
| S1.9 | 24.4 | 24.4 | 23.87 | 24.4 |

In the unlimited case. Metaheuristic: 10 times by 1 second
Gurobi: 10 minutes running time limit.

# Conclusions

- A new robust transfer line balancing problem with limited exctra processing times is considered.
- A calculation of the stability radius is provided and validated.
- A parallel metaheuristic for maximization of the stability radius is developed.
- It would be worthwhile to extend this approach to the case, where uncertain tasks have different time limits.

Thank you for your attention!