Эволюционные алгоритмы для задачи составления энергетически эффективного расписания на одном процессоре

Юлия Викторовна Захарова

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Институт математики им. С.Л. Соболева

Report structure

- Problem Statement
- Previous Research and Our Results
- Local Search
- Evolutionary Algorithms
- Computational Experiment

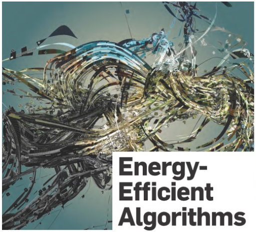
$1|r_j, d_j|E$

Input Data

$J = \{1, ..., n\}$ is the set of jobs.
W_j is the volume of job j.
r_j is the release date of job j.
d_j is the deadline of job j.
Preemptions are disallowed.

Agreeable Release Dates and Deadlines

For any two jobs i and j, relation $r_i < r_j$ implies $d_i \leq d_j$.

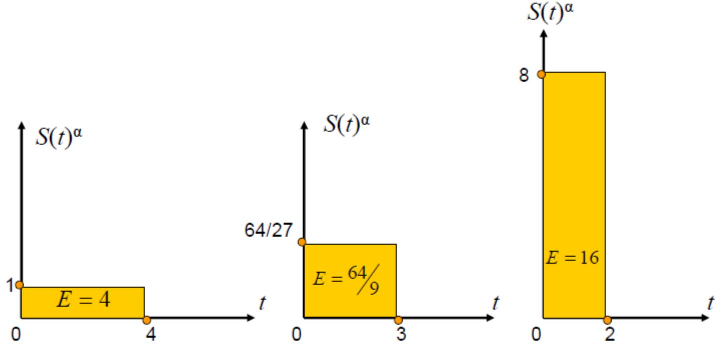


Homogeneous Model in Speed-scaling

If a processor runs at speed s then the energy consumption is s^α units of energy per time unit, where $\alpha > 1$ is a constant (practical studies show that $\alpha \leq 3$).

It is supposed that a continuous spectrum of processor speeds is available.

The objective is to find a feasible schedule that minimizes the total energy consumption.



Related Results: Algorithms

Energy-Efficient Scheduling for Parallel Real-Time Tasks Based on Level-Packing

Kong F. et. al. (SAC'11): two-dimensional strip packing problem, energy consumption assignment

Energy efficient scheduling of parallel tasks on multiprocessor computers

Li K. (Journal of Supercomputing, 2012): system partitioning, task scheduling, power supplying

A genetic algorithm for energy-efficiency in job-shop scheduling

Salido M.A. (Int. J. Adv. Manuf. Technol., 2015): genetic algorithm with generational scheme – position-based encoding, problem specific initial population, order crossover, shuffle mutation.

Related Results: Complexity

$1|pmtn, r_j, d_j|E$ and $P|agree, r_j, d_j|E$

Yao, Demers, Shenker (1995): $O(n^2)$ time;
Shioura, Shakhlevich, Strusevich (2015): $O(n^3)$ time.

$1|r_j, d_j|E$

Antoniadis, Huang (2013): NP-hard, $2^{5\alpha-4}$;
Bampis, Kononov, Letsios et. al. (2018): $2^{\alpha-1}(1+\varepsilon)^{\alpha}\tilde{B}^{\alpha}$.

Preemptive and Agreeable instances

Algorithm 1 YDS Algorithm (Yao, Demers, Shenker), 1995

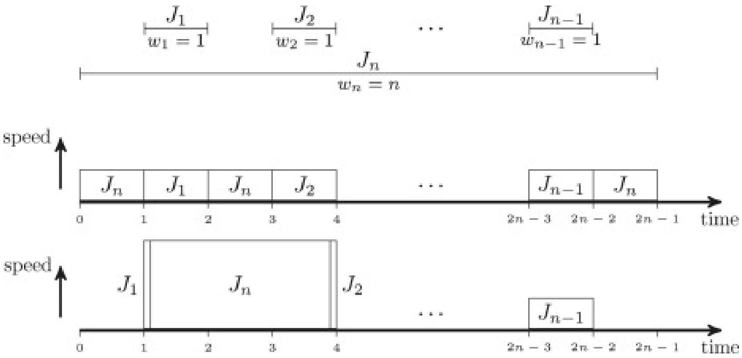
1: While $\mathcal{J} \neq \emptyset$:

1.1 Let $[t, t')$ be the interval with maximum density, i.e., that maximizes $\frac{\sum_{j \in J(t,t')} W_j}{t'-t}$.

1.2 Process jobs $i \in J(t, t')$ in interval $[t, t')$ using the earliest deadline policy with speed equal to the maximum density. Then remove the jobs $J(t, t')$ from $\mathcal{J}$, and adjust the remaining jobs as if the time interval $[t, t')$ does not exist.

2: Return the resulting schedule and its objective value.

Preemptive vs Non-Preemptive



Small Neighborhoods

Solution encoding

Solutions are encoded as permutations.
Consider a pair of indexes $i < j$ and correct release dates and deadlines as follows: $r'_j = \max\{r_i,\ r_j\}$ and $d'_i = \min\{d_i,\ d_j\}$.
Objective value may be calculated in $O(n^2)$ time for the given permutation.

Neighborhoods

Swap neighborhood: positions of two jobs are exchanged.
Insert neighborhood: inserting a job in some other position.

Partial Order Between Jobs

Release dates and deadlines give us a partial order between jobs: if $d_i < r_j$ then job i must precedes job j.
We exchange only independent jobs in the neighborhoods.

Large Neighborhoods

Optimal Recombination Problem (ORP)

Given two parent solutions π^1 and π^2. It is required to find a permutation π' such that:

(I) $\pi'_i = \pi^1_i$ or $\pi'_i = \pi^2_i$ for all $i = 1, \ldots, n$;

(II) π' has the minimum value of objective function $E(\pi')$ among all permutations that satisfy condition (I).

Optimal recombination may be considered as a best-improving move in a large neighbourhood defined by two parent solutions.
The ORP is NP-hard, but "almost all" instances are polynomially solvable.

Population Local Search (PLS)[1] [NUMTA2023]

1: Construct the initial population of m permutations (feasible in accordance with release dates and deadlines).
2: Apply local search based on swap or insert neighborhood to each permutation.
3: For j=1 to m perform Steps 4-6:
4: Generate random sequence of permutations $\pi^1, \ldots, \pi^m$.
Put $\pi' = OR(\pi^1, \pi^2)$.
5: For i=3 to m construct

$$\pi' = OR(\pi', \pi^i).$$

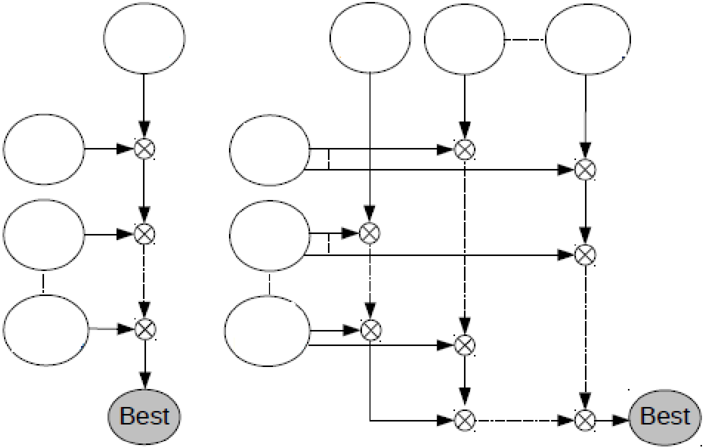
6: Improve π' by local search withing swap or insert neighborhood and save as π'_j.
7: Put $\pi'' = OR(\pi'_1, \pi'_2)$.
8: For j=3 to m construct

$$\pi'' = OR(\pi'', \pi'_j).$$

9: Return π'' and $E(\pi'')$.

[1] R. Tinos, D. Whitley, G. Ochoa (2020): A New Generalized Partition Crossover for the Traveling Salesman Problem: Tunneling Between Local Optima

Population Local Search (PLS) [NUMTA2023]



Genetic Algorithm with OR ($GA : ORP$) [NUMTA2023]

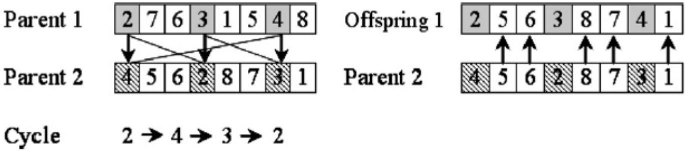
1: Construct the initial population of m permutations (feasible in accordance with release dates and deadlines).
2: Apply local search based on swap or insert neighborhood to each permutation.
3: Until termination condition is met, perform

- 3.1 Select two parent permutations π^1 and π^2.
- 3.2 Apply swap or insert mutation to permutations π^1 and π^2.
- 3.3 Put $\pi' = OR(\pi^1, \pi^2)$.
- 3.4 Replace the worst permutation of the population by π'.

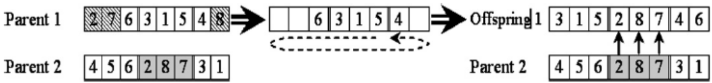
4: Improve the record solution by local search withing swap or insert neighborhood.
5: Return the best found solution.

Crossover Operators

Cycle Crossover (CX)

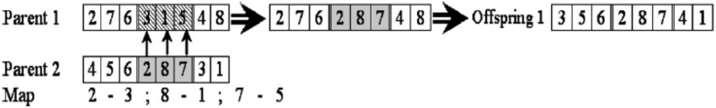


Order Crossover (OX)

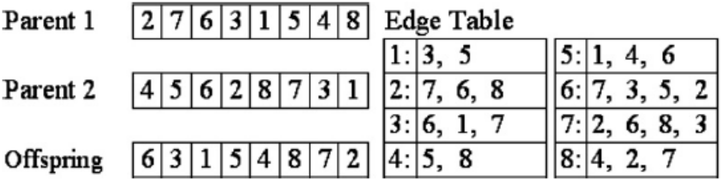


Crossover Operators

Partially Mapped Crossover (PMX)



Edge Recombination (EX)



Genetic Algorithm with Generational Scheme (GAgs)

1: Construct the initial population of m permutations.
2: Apply local search based on swap or insert neighborhood to each permutation.
3: Until termination condition is met, perform
for $i \leftarrow 1$ to βm

   2.1 Select two parent permutations π^1 and π^2.

   2.2 Construct $(\pi^{1\prime}, \pi^{2\prime}) = Cross(\pi^1, \pi^2)$.

   2.3 Apply insert mutation to permutations $\pi^{1\prime}$ and $\pi^{2\prime}$.

   2.4 Compute the objective value of the offspring.

4: Return the best found solution.

Adaptive Technique

$$\phi_a = \begin{cases} w_1, \text{if the new solution is a new global best,} \\ w_2, \text{if the new solution is better than the current one,} \\ w_3, \text{if the new solution is accepted,} \\ w_4, \text{if the new solution is rejected.} \end{cases}$$

$$\rho_a = \lambda \rho_a + (1 - \lambda)\phi_a.$$

Computational Experiment: Input Data

Number of jobs $n = 50$ and $n = 100$.

Parameter $\alpha = 2$ and $\alpha = 3$.

Release date r_j is selected randomly from interval $[0,\ 20]$.

Deadline d_j is generated randomly from $[r_j + 1,\ r_j + 11]$.

Volume W_j is chosen randomly from $[5,\ 15]$.

Computational Experiment: Relative Deviation from Lower Bound

Series	*PLS*			*GA : ORP*		
	min	aver	max	min	aver	max
$S_{\alpha=2},\ n{=}50$	0	**0,9**	6,1	0	**1,7**	8,2
$S_{\alpha=3},\ n{=}50$	0	**2,1**	10,1	0	**3,2**	12,7
$S_{\alpha=2},\ n{=}100$	1,7	**3,1**	6,1	2,8	**5,2**	7,9
$S_{\alpha=3},\ n{=}100$	0,5	**5,8**	15,1	0,8	**8,8**	17,3

Series	*GA : Adapt*			*GA : PMX*		
	min	aver	max	min	aver	max
$S_{\alpha=2},\ n{=}50$	0	**0,6**	3,1	0	**0,7**	3,7
$S_{\alpha=3},\ n{=}50$	0	**1,9**	9,5	0	**2,0**	9,7
$S_{\alpha=2},\ n{=}100$	1,5	**3,1**	5,5	1,7	**3,1**	5,9
$S_{\alpha=3},\ n{=}100$	0,3	**5,7**	12,5	0,3	**5,9**	12,3

Series	*GA : Mut*		
	min	aver	max
$S_{\alpha=2},\ n = 50$	0	**1,9**	9,0
$S_{\alpha=3},\ n = 50$	0	**3,9**	9,8
$S_{\alpha=2},\ n = 100$	3,2	**6,0**	9,5
$S_{\alpha=3},\ n = 100$	2,1	**9,1**	12,7

Conclusion and Further Research

- We proposed and investigated the evolutionary algorithm with various operators and schemes for the single processor speed scaling scheduling problem.
- Experimental evaluation on instances of different structures shown that the algorithms demonstrate competitive results.
- Further research can be undertaken to various optimized and/or randomized recombination operators and comparison of them in context of the presented algorithms and their composition; generalize to the case of several processors.

Thank you for your attention!