FINITE-DIFFERENCE METHOD FOR CALCULATING A TWO-DIMENSIONAL LAMINAR FLAME

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The present study will examine propagation of a laminar flame in the gap between two parallel plates, the temperature of which is specified. Two models will be considered (with and without diffusion of the mixture in the vertical direction) and a comparative numerical analysis of the models will be performed. A similar problem was considered in [1, 2]. In [1], the parabolized system of heat and mass transfer equations was solved simultaneously with the equations of motion and continuity by the iteration-interpolation method. However, the effect of channel width and Lewis number Le upon combustion was not analyzed, which is important in constructing mixture igniters for combustion chambers.

We will consider the equations describing the process of free propagation of a laminar flame between two parallel plates in a Cartesian coordinate system fixed to the flame front [3]:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}-\frac{Gc_p}{\lambda}\frac{\partial T}{\partial x}=\frac{H_c}{\lambda}w,$$
$$\mathrm{Le}\,\frac{\partial^2\alpha}{\partial x^2}-\frac{Gc_p}{\lambda}\frac{\partial\alpha}{\partial x}=-\frac{c_p}{Y_0\lambda}w, \qquad (1)$$

where T is the temperature of the stoichiometric mixture; α, relative fuel concentrations; x, y, directed along and across the gap, c_p, specific heat; λ, thermal conductivity coefficient of the mixture; Y_0, initial mass concentration of fuel; H_c, heat of combustion; w, velocity of the single-step chemical reaction, defined by the Arrhenius law; $G = \rho u$ = const, mixture flow rate; ρ, density; u, mixture velocity along the x axis. Since the combustion process is symmetrical, we will consider only one-half of the gap width. Then the boundary conditions for system (1) take on the form

$$\begin{aligned} x=-\infty:&\quad T=T_0,\ \alpha=1,\\ x=+\infty:&\quad \partial T/\partial x=\partial\alpha/\partial x=0,\\ y=0:&\quad T=T_0,\\ y=L:&\quad \partial T/\partial y=0. \end{aligned} \qquad (2)$$

We note that in numerical analysis of the problem of Eqs. (1) and (2), it is necessary to take a sufficiently large integral [0, a] along the x axis. We will study the effect of Lewis number. Let $\alpha_0(x, y)$ be a solution of Eqs. (1) and (2) at Le = 0. On the basis of asymptotic analysis of [3], we conclude that

$$\alpha(x, y)=\alpha_0(x, y)+0(\mathrm{Le\,Pe}^{-1}),\ \mathrm{Pe}=Gc_p/\lambda. \qquad (3)$$

This representation of the function $\alpha(x, y)$ has important significance for specifying the initial concentration field and evaluating its behavior in the limiting case.

We define a solution of Eqs. (1) and (2) by Picar linearization with subsequent application of finite-difference methods to the linearization steps. The mixture flow rate is calculated as the eigenvalue of the problem for each iteration in nonlinearity [3]. Numerical comparison of central directed Samarskii difference methods [4] and the exponentially driven method of [5, 6] shows that it is desirable to use the latter if the boundary conditions are specified such that there is a temperature boundary layer with respect to the number Pe. In the opposite case, satisfactory accuracy is produced by Samarskii and directed difference methods.

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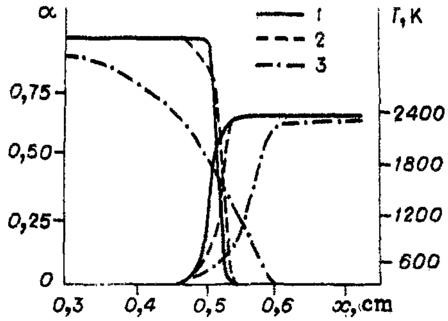

Fig. 1

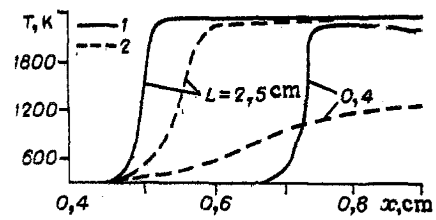

Fig. 2

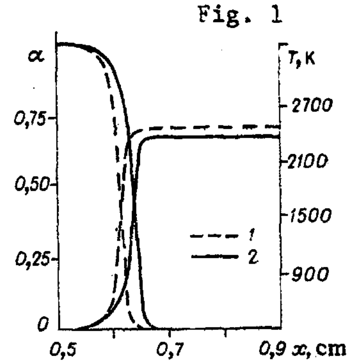

Fig. 3

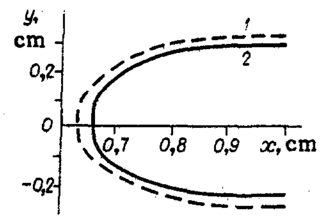

Fig. 4

Fig. 1. Functions T(x, L) and α(x, L) at Le' = 0 (1), 10^{-4} (2), and 10^{-3} (3).

Fig. 2. Temperature vs interplate distance at Le' = 0 (1) and 10^{-3} (2).

Fig. 3. Functions T(x, L) and α(x, L) with (1) and without (2) consideration of vertical diffusion.

Fig. 4. Flame front with (1) and without (2) consideration of vertical diffusion.

TABLE 1

Le'	G	N
0	0,57	8
10^{-5}	0,52	20
10^{-4}	0,36	13
10^{-3}	0,18	12

TABLE 2

L	G	N
0,05	0,56	9
0,025	0,57	8
0,01	0,56	11
0,008	0,54	35
0,006	0,47	31
0,004	0,45	108
0,001	0,00	6

We will consider results of numerical experiments on modeling combustion of a propane–air mixture [3]. Figure 1 shows graphs of T(x, L) and α(x, L) for various values of Le' = Le/Pe, which confirm Eq. (3). Table 1 shows values of G and the number of iteration (N) in nonlinearity as function of Le'.

We will analyze the dependence of T and α on interplate distance. Table 2 shows the dependence of flow rate and number of iterations on L, if Le = 0. From analysis of the results shown therein, it can be concluded that G decreases abruptly as $L \to L_{cr}$ (where L_{cr} is the interplate distance at which combustion terminates), while the number of iterations in nonlinearity increases. It was assumed in [2] that L_{cr} = 0.1281 cm at Le = 0, which agrees with the data of Table 2.

The dependence of temperature on interplate distance is shown in Fig. 2. Calculations showed that L_{cr} is independent of Le. If Le' = 10^{-3} and L = 0.15 cm, extinction of the flame occurs, in contrast to [2], in which burning did not cease at Le = 0. Analysis of Table 1 reveals that the value of G decreases with increase in Le, which indicates a dependence of reaction rate on diffusion processes in the reacting gas.

We will turn from the model of Eqs. (1) and (2) to a more general one considering mass transport in the vertical direction by means of diffusion. Following Fick's law [7], we express the mixture velocity in the vertical direction in the form $v = D/\alpha \cdot \partial\alpha/\partial y$ and write equations for the combustion process in the diffusion approximation

$$\begin{gathered}\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}-\frac{Gc_p}{\lambda}\frac{\partial T}{\partial x}-\frac{\mathrm{Le}}{\alpha}\frac{\partial\alpha}{\partial y}\frac{\partial T}{\partial y}=\frac{H_c}{\lambda}w,\\ \mathrm{Le}\frac{\partial^2\alpha}{\partial x^2}+\mathrm{Le}\frac{\partial^2\alpha}{\partial y^2}-\frac{Gc_p}{\lambda}\frac{\partial\alpha}{\partial x}-\frac{\mathrm{Le}}{\alpha}\left[\frac{\partial\alpha}{\partial y}\right]^2=-\frac{c_p w}{Y_0\lambda}.\end{gathered}\tag{4}$$

To boundary conditions (2) we add the condition $\partial\alpha/\partial y = 0$ at $y = 0; L$. In the numerical solution of system (4) and (2), a grid containing 40 nodes along x and 20 along y was used. The y-grid was constructed nonuniformly so that a portion of the nodes fell within the flame front. Figure 3 shows graphs of $T(x, L)$ and $\alpha(x, L)$ with and without consideration of vertical diffusion. It follows from analysis of Fig. 3 that in the case of the model of Eqs. (4) and (2), combustion occurs more actively with the reaction rate and adiabatic temperature increasing. Figure 4 shows the flame front for the cases indicated.

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PYROLYSIS OF TWO-LAYER THERMOPROTECTIVE MATERIAL UNDER THE ACTION OF A SPECIFIED HEAT FLUX

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Mathematical modeling of the thermochemical destruction and combustion of coking thermoprotective material (TPM) was discussed in [1-4]. In the present work, the thermochemical destruction of a two-layer thermoprotective coating with a first layer consisting of Teflon or capron and filler and a second layer of glassfiber is considered theoretically. The thermochemical destruction of the two layers is described using the model of nondeformed or porous reacting body [4] and significantly different thermophysical and thermokinetic constants characterizing the structure and reaction properties of the TPM. The influence of mechanical entrainment on the characteristics of breakdown and combustion is investigated.

Formulation of the Problem

With the aim of increasing the effectiveness of TPM, a protective film (quasisublimator) is often applied to the surface. In the present problem, the external (first) dispersive layer is taken to be a quasisublimator (thickness l_1) consisting of Teflon or capron and particles of inert filler (Sb_2O_3). The second (internal) layer is coking glassfiber of finite dimension (l_2). It is assumed that the variable convective heat flux $q_w(t)$ or radiant flux of constant intensity q_r acting on the TPM for a definite time is specified. It is necessary to find the temperature and pressure fields of the gas at any depth, surface temperature, mass entrainments, and linear rate of failure-surface motion as a function of the time.

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