FUZZY MODELS OF "COSTS" AND "PROFIT"

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This article describes the construction of fuzzy models "Costs" and "Profit" for operational decisions in the economic system management by introducing linguistic variables and application of expert rules.

Keywords: model, linguistic variable, costs, profit, expert analysis.

Introduction

Profit reflects the effectiveness of the company and it shows how the sales proceeds of product (goods and services) exceed the production costs. In a free market economy, the company's management takes his own decisions and plans volume of production, so the modelling of a "Costs-Profit" system is an actual problem, because it allows to develop an effective production plan and pricing of products.

As part of the research work "Application of information technology for the economic system characteristics calculation using fuzzy logic" we developed the "Costs" and "Profit" models which were applied on a real company which sells hardware and software products and performs services in IT. The models are constructed using linguistic variables that approximate human reasoning.

Costs and profit as characteristics of an economic system

The free market economy orients enterprises to serve market demand and specific requests of customers. Focusing the manufacturing processes to products types that are in demand is necessary to have profits for the company. The market is characterized by a constant desire to improve production efficiency. It implies freedom of decision-making by the people who are responsible for the results of the company and its subsidiaries; it requires constant adjustment of goals and planning programs depending on market conditions. This requires a special control system, typical for market conditions and priorities and mentality of Russian business, because many Russian companies underestimate costs in accounting and it implies significant economic losses for their activity.

The scientific method of costs classification is of great importance for the correct organization of accounting. Production costs are grouped by their places of origin, centres of responsibility, types of expenses and their sources. Fixed and variable costs are used for break-even analysis and its related indicators, and for production optimization. Variable costs vary in proportion to production or sales volumes, whereas costs for a single unit of output are constant.

Fixed costs in total do not vary with the level of business activity, but they decrease with volume of production or sales increasing, if they are calculated to the unit of production.

As for profits, there are usually gross (balance sheet, total) and net profit calculated, remaining after gross profits taxes and other deductions.

The company where this research has been applied mostly works with corporate customers. Prices of hardware, software and customer services are determined depending on various factors. For example, the price of an accounting system will be formed as follows: the costs plus 20-25% (depending on total costs and the client financial capabilities).

In a free market system there is a dynamic interaction between profits and costs. Their values, as well as value of others attributes, can be considered as quantitative and qualitative variables using the tools of fuzzy logic.

Formalization of the profit management procedure

For calculating profit growth rates we suggest to use the linguistic variable

$$C=x, \ Tx, \ X, \ G, \ M,$$

where x is the variable name;
$T(x)$ is the set of names (terms) of the linguistic values x, each of which is a fuzzy set on the set X;
G is a syntactic rule for new terms formation with the ligaments of "and", "or", and localizing modifiers clarifying values of individual characteristics, like "very," "almost," etc.;
M is the semantic rule for associating each value with its concept [1].

We propose to introduce two linguistic variables: "Costs" and "Profit". The linguistic variable "Costs" has the terms "small", "medium", "large"; the values of the linguistic variable "Profit" are chosen between the terms of "low", "medium", and "high".

There are several methods for constructing fuzzy set membership functions by expert estimates. Using direct methods the expert directly defines the rules for determining values of the membership function $\mu_A(x)$ which characterize the element x. Usually, the direct methods are used to describe the concepts that have measurable properties, such as height, weight, volume. In this case, the direct definition of the membership function values is more appropriate. With indirect methods the values of membership functions are chosen by preselected conditions. In that case the expert input is treated as raw data suitable for further processing. Additional conditions may be imposed to the form of received information, as well as to the processing procedure [2].

An indirect method for constructing membership functions based on rank estimates

Indirect methods for determining the values of the membership functions are used in cases when there are no basic measurable properties to define a fuzzy set. This method has been developed by A. P. Rothstein, and it is based on the idea of distribution of membership values of universal set elements according to their ranks. The rank of an element $x_i \in X$ is the value $r_s(x_i)$ which is a

characterization of importance of this element in the formation of the properties in fuzzy term S [3]. Let's assume the following rule: the higher the rank of an element, the greater the degree of membership. Also let's introduce the notation $r_s(x_i)=r_i$, $\mu_s(x_i)=\mu_i$, $i=(1,n)$. We are going to use a 9-point scale (1 is the lowest rank and 9 is the highest rank) for expert reviews.

Let's take four values for the linguistic variable "Costs": $x1=1000$, $x2=18000$, $x3=45000$, $x4=50000$. The ratios of ranks to these elements for the fuzzy terms of "small", "medium", "large" are showed in the table 1.

Table 1.

	small	medium	large
r1	9	3	1
r2	3	9	2
r3	1	2	5
r4	1	1	9

The next step of determination of the membership degree $\mu_s(x_i)$ is the calculation of relative score of ranks, which form the matrix:

$$A=\begin{bmatrix} 1 & \dfrac{r_2}{r_1} & \dfrac{r_3}{r_1} & \cdots & \dfrac{r_n}{r_1} \\[2mm] \dfrac{r_1}{r_2} & 1 & \dfrac{r_3}{r_2} & \cdots & \dfrac{r_n}{r_2} \\[2mm] \cdots & \cdots & \cdots & \cdots & \cdots \\[2mm] \dfrac{r_1}{r_n} & \dfrac{r_2}{r_n} & \dfrac{r_3}{r_n} & \cdots & 1 \end{bmatrix}$$

The calculations are performed for all the terms of the linguistic variable "Costs." The matrices of these ranks are displayed in the Table 2.1-2.3.

Table 2.1.

small	x_1	x_2	x_3	x_4
x_1	1	0.33	0.11	0.11
x_2	3	1	0.33	0.33
x_3	9	3	1	1
x_4	9	3	1	1

Table 2.2.

medium	x_1	x_2	x_3	x_4
x_1	1	3	0.66	0.33
x_2	0.33	1	0.22	0.11
x_3	1.5	4.5	1	1
x_4	3	9	2	1

Table 2.3.

large	x_1	x_2	x_3	x_4
x_1	1	2	5	9
x_2	0.5	1	2.5	4.5
x_3	0.2	0.4	1	1
x_4	0.11	0.22	0.55	1

Using the data obtained, according to the conditions $\mu_1+\mu_2+\ldots+\mu_n=1$ and $\mu_{n-1}=(r_{n-1}/r_n)\mu_n$, we have founf the elements of the membership function for each term (values on the assessments scale):

$$\mu_1 = \left(1 + \frac{r_2}{r_1} + \frac{r_3}{r_1} + \ldots + \frac{r_n}{r_1}\right)^{-1}$$

$$\mu_2 = \left(\frac{r_1}{r_2} + 1 + \frac{r_3}{r_2} + \ldots + \frac{r_n}{r_2}\right)^{-1}$$

....

$$\mu_n = \left(\frac{r_1}{r_n} + \frac{r_2}{r_n} + \frac{r_3}{r_n} + \ldots + 1\right)^{-1}$$

The calculated values of the membership function are displayed in the table 3.

Table 3.

	small	medium	large
1000	0.642857	0.2	0.0588235
18000	0.214286	0.6	0.1176471
45000	0.071429	0.125	0.3846154
50000	0.071429	0.0666667	0.5294118

The obtained values of membership functions (the vertical axis) in normalized form are shown in the chart (Chart 1). On the horizontal axis there are prices in roubles.
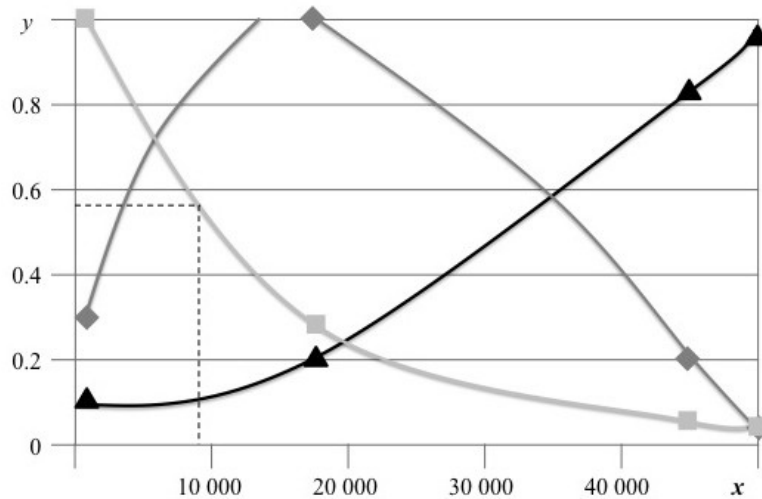

Chart 1. The model "Costs" (■ - "small", ◆ - "medium", Δ - "large")

For calculating the values of the membership function for the linguistic variable "Profit" we used a similar procedure. For the next values of variable x: x1=1000, x2=6000, x3=12000, x4=50000, we obtained the relations between the elements and ranks in fuzzy terms which are showed in the table 4.

Table 4.

	low	medium	high
r1	9	1	1
r2	1	9	1
r3	1	2	3
r4	1	1	9

The values are obtained for the rank matrices are shown in the tables 5.1-5.3.

Table 5.1.

low	x_1	x_2	x_3	x_4
x_1	1	0.11	0.11	0.11
x_2	9	1	1	1
x_3	9	3	1	1
x_4	9	3	1	1

Table 5.2.

medium	x_1	x_2	x_3	x_4
x_1	1	9	2	1
x_2	0.11	1	0.22	0.11
x_3	0.5	4.5	1	0.5
x_4	1	9	2	1

Table 5.3.

high	x_1	x_2	x_3	x_4
x_1	1	1	3	9
x_2	1	1	3	9
x_3	0.33	0.33	1	3
x_4	0.11	0.11	0.33	1

Also the elements of membership functions for all terms have been found (Table 6).

Table 6.

	low	medium	high
1000	0.75	0.0769231	0.0714286
6000	0.083333	0.6923077	0.0714286
12000	0.083333	0.1538462	0.2142857
50000	0.083333	0.0769231	0.6428571

The membership function (the vertical axis) in the normalized form is shown in the next chart (Chart 2). The horizontal axis indicates the price in roubles.
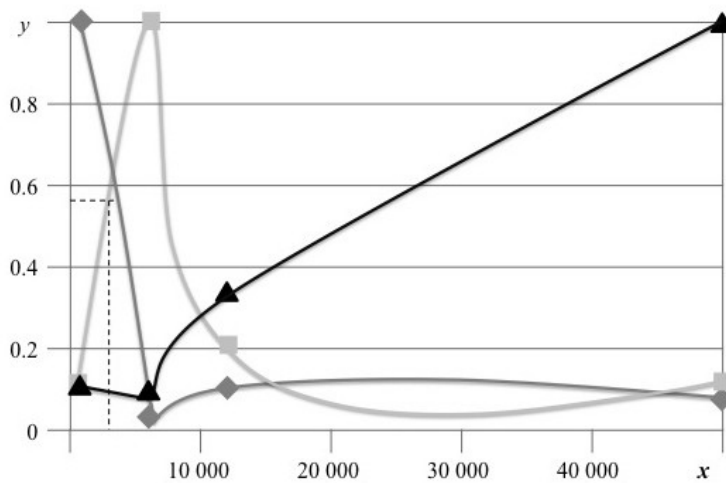
Chart 2. The model "Profit" (◆ - "low", ◼ - "medium", Δ - "high")

An example.

Let the cost of software installation be 9000 roubles. Then, according to the "Costs" chart (Chart 1), the membership degree of the term "small" on a scale of assessments will be 0.55. We apply the rule "low costs correspond to low profit". Accordingly, on the graph "Profit" (Chart 2) the intersection of the term "low" with the estimate of 0.55 corresponds to a value of about 3000 roubles.

Currently this method of costs and profits calculation is applied within Prime LLC, which sells hardware, software and IT services to its customers.

Conclusion

In many practical situations, the operating decisions that are based on approximate reasoning can be quite successful. That results enable acquiring the necessary information about economic system characteristics and making informed decisions.

Our results were highly appreciated by the administrative personnel of the company and they were found quite sufficient for operational management of the current production process.

Bibliography

[1] L.A. Zadeh, Fundamentals of a new approach to the analysis of complex systems and decision-making processes // Mathematics Today. – Mir, Moscow, 1974, №7, pp. 5-49.

[2] V.A. Marenko, O.N. Luchko, The modeling of cost management using fuzzy logic // Devices and Systems. Management, control, diagnostics, 2009, №12, pp. 45-47.

[3] A.P: Rothstein, Intelligent identification technology: fuzzy logic, genetic algorithms, neural networks. – Vinnitsa UNIVERSUM, Vinnitsa, 1999, pp. 320.

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